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B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2022

Fourth Semester

Complementary Course—Mathematics

FOURIER SERIES, DIFFERENTIAL EQUATIONS, NUMERICAL ANALYSIS AND ABSTRACT ALGEBRA

(For the programme B.Sc. Physics/Chemistry/Petrochemicals/Geology/Food Science and Quality Control and Computer Maintenance and Electronics)

(2013–2016 Admissions)

Time : Three Hours | Maximum Marks : 80

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Define Fourier series for a periodic function $f(x)$ with period $2\pi i$. Also define the Fourier co-efficients.

2. Are the following function even, odd or neither even nor odd $|x|, x+x^2$.

3. Find the radius of convergence of the series $\sum_{m=0}^{\infty} \frac{(-1)^m}{8^m} x^{3m}$.

4. Define the Legendre polynomial of degree n.

5. Find the order and degree of the partial differential equation :

$$\left(\frac{\partial z}{\partial x}\right)^2 + \frac{\partial^3 z}{\partial y^3} = 2x\left(\frac{\partial z}{\partial x}\right).$$

6. Give an example each of a non-linear and linear partial differential equation.

7. If X = 0.51 and is correct to two decimal places, then find the absolute accuracy and relative accuracy.

8. Define a group.

9. What is the order of the cyclic subgroup of Z_4 generated by 3.

10. Define homomorphism and isomorphism of rings.

$(10 \times 1 = 10)$

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Write the Legendre's polynomials :

    $p_0(x), p_1(x), p_2(x)$ and $p_3(x)$.

12. Define Gamma function. Find $\Gamma(1)$ and $\Gamma(2)$.

13. Find the radius of convergence of the series $\sum_{m=0}^{\infty} \frac{(2m)!}{(2m+2)(2m+4)} x^m$.

14. Find a partial differential equation by eliminating the arbitrary constants a and b from $z = (x-a)^2 + (y-b)^2$.

15. Find the partial differential equation by eliminating the arbitrary function f from the equation $z = f(xy/z)$.

16. Two numbers are given as 2.5 and 48.289, both of which being correct to the significant figures given. Find their product.

17. Find the quotient $q = x/y$, where $x = 4.536$ and $y = 1.32$, both x and y being correct to the digits given. Find also the relative error in the result.

18. In the Maclaurin series expansion for e^x find n the number of terms such that their sum yields the value of e^x correct to 8 decimal places at $x = 1$.

19. Show that the inverse element is unique for each element a in a group G.

20. Describe all the elements in the cyclic subgroup of $GL(2,\mathbb{R})$ generated by the 2×2 matrix $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$.

21. Compute $\ker(\phi)$ and $\phi(25)$ for $\phi : z \to z_7$ such that $\phi(1) = 4$.

22. Determine whether :

    $\{(-1,1,2), (2,-3,1), (10,-14,0)\}$ is a basis for α^3 over $\mathbb{R}$.

(8 × 2 = 16)

Part C (Short Essays)

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. Find the Fourier series for the function $f(x)=\begin{cases}1 \text{ if } 0 \le x \le \pi \\ 0 \text{ if } 0-\pi < \pi < 0\end{cases}$ with period 2π.

24. Solve by power series method the equation $y' + xy = 0$.

25. Show that $J_0^1(x) = -J_1(x)$.

26. Solve the equation $a(p+q) = z$.

27. By false position method, obtain a root correct to three decimal places of the equation $x^3 - x^2 - 1 = 0$.

28. By bisection method obtain a root of $x^3 - 4x - 9 = 0$, correct to 3 decimal places.

29. Let A be a non-empty set and let S_A be the collection of all permutations of A. Show that S_A is a group under permutation multiplication.

30. Describe all ring homomorphisms of Z into Z × Z.

31. Prove that intersection of subspaces of a vector space V is again a subspace of V over F.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Find the Fourier series for the function $f(x) = x^2, -1 < x < 1$, with period $p = 2L = 2$.

    (b) Find the Fourier sine and cosine series for the function $f(x)=\begin{cases}0, & 0 < x < 2 \\ 1, & 2 < x < 4\end{cases}$.

33. Determine the surface which satisfies the differential equation :

$$\left(x^2 - a^2\right)p + (xy - az\tan\alpha)q = xz - ay\cot\alpha$$ and passed through the curve $x^2 + y^2 = a^2, z = 0$.

Turn over

34. (a) Use Newton-Raphson method to obtain a root correct to 3 decimal places of the equation $x + \log x = 2$.

(b) Use quotient difference method to obtain the approximate roots of the equation $x^3 - 7x^2 + (0) - 2 = 0$.

35. (a) Let G be a cyclic group with generator a. Prove that if the order of G is infinite, then G is isomorphic to (Z, +). If G has finite order n, prove that G is isomorphic to $(Z_n, +_n)$.

(b) Prove that is a finite dimensional vector space, every finite set of vectors spanning the space contains a subset that is a basis.

(2 × 15 = 30)