E 3719

00003719

Reg. No.................................

Name.....................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2022

Fourth Semester

Complementary Course—OPERATIONS RESEARCH—NON-LINEAR PROGRAMMING

(For B.Sc. Mathematics—Model II)

[2013—2016 Admissions]

Time : Three Hours   Maximum Marks : 80

Part A

Answer **all** *questions.*

Each question carries 1 *mark.*

1. What is integer programming ?
2. What is the difference between integer programming and mixed integer programming ?
3. What do you mean by branch and bound technique ?
4. Define 0-1 programming.
5. What is a Lagrangian function ?
6. Write the matrix form of general non-linear programming problem.
7. Define Saddle point.
8. Define quadratic programming problem.
9. Define positive semi definite quadratic form.
10. Define a separable programming problem.

(10 × 1 = 10)

Part B

Answer any **eight** *questions.*

Each question carries 2 *marks.*

11. What is the difference between integer programming and linear programming.
12. Explain Gomory's cutting plane algorithm.

Turn over

13. Explain the applications of integer programming models.

14. Write the necessary conditions for non-negative saddle point.

15. Show that $f(X) = 2x_1^2 + x_2^2$ is a convex function over all of R^2.

16. Give an example each of a positive definite and negative definite quadratic form.

17. Show that if $X^T QX$ is positive semi-definite, then it will be convex for all $X \in R^4$, where Q be a symmetric $n \times n$ real matrix.

18. Split $f(x) = 5x_1^2 + x_2^2 - 3x_1 + 5x_2$ as the sum of two convex functions.

19. Consider the problem :

    Minimize $Z = x_1^2 + 2x_2^2 - 2x_1$

    subject to the constraints

    $$x_1^2 + x_2^2 \leq 4, x_1, x_2 \geq 1.$$

    Is this a convex programming problem.

    If not how will you proceed to solve this problem.

20. Define a separable function. Give an example.

21. Write the necessary conditions for non-negative saddle points.

22. Explain Kuhn-Tucker conditions.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. Find the optimum integer solution to the problem, Maximize Z = $x_1 + 2x_2$

    subject to $5x_1 + 7x_2 \leq 21$

    $-x_1 + 3x_2 \leq 8$

    x_1, x_2 being non-negative integers.

24. Explain the steps of branch and bound algorithm.

25. Find the optimal solution to the following integer programming problem using Gomory's algorithm :

Maximize $Z = x_1 - x_2$

subject to the constraints :

$x_1 + 2x_2 \leq 4$

$6x_1 + 2x_2 \leq 9$

$x_1, x_2 \geq 0$, x_1, x_2 are integers.

The optimal simplex table corresponding to the above problem is given below :

x_{13}	x_1	x_2	x_3	x_4
$x_1 = 3/2$	1	2/6	0	1/6
$x_2 = 5/2$	0	10/6	1	–1/6
$z = 3/2$	0	8/6	0	1/6

x_3, x_4 are slack variables.

26. Obtain the necessary and sufficient conditions for the optimum solution of the following non-linear programming problem :

Minimize $Z = f(x_1, x_2) = 3e^{2x_1 + 1} + 2e^{x_2 + 3}$

subject to the constraints : $x_1 + x_2 = 7$

and $x_1, x_2 \geq 0$.

27. Solve by using the method of Lagrangian multiplier, the problem :

Minimize $Z = 6x_1^2 + 5x^2$

subject to the constraints :

$x_1 + 5x_2 = 3$

$x_1, x_2 \geq 0$.

Turn over

28. Explain the role of Lagrange multipliers in a non-linear programming problem.

29. How does a quadratic programming differ from a linear programming problem. Explain.

30. Solve by cutting plane method :

    Maximize $Z = 7x_1 + 9x_2$

    subject to the

    $$-x_1 + 3x_2 \leq 6$$

    $$7x_1 + x_2 \leq 35$$

    $x_1, x_2 \geq 0$, and x_1 is an integer.

31. Solve graphically the non-linear programming problem :

    Minimize $Z = (x_1 - 1)^2 + (x_2 - 2)^2$

    subject to the constraints :

    $$0 \leq x_1 \leq 2, 0 \leq x_2 \leq 1.$$

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. Use Branch and Bound method to solve the integer programming problem :

    Maximize $Z = 3x_1 + 4x_2$

    subject to the constraints :

    $$3x_1 - x_2 + x_3 = 12$$

    $$3x_1 + 11x_2 + x_4 = 66$$

    $$x_j \geq 0, \; j = 1, 2, 3, 4.$$

33. Use Wolfe's method to solve the quadratic programming problem :

Maximize $Z = 8x + 10x_2 - x_1^2 - x_2^2$

subject to $3x_1 + 2x_2 \leq 6,$

$x_1, x_2 \geq 0.$

34. Use separable convex programming to solve the problem :

Maximize $f(x) = 3x_1 + 2x_2$

subject to the constraints

$g(x) = 4x_1^2 + x_2^2 \leq 16,$

$x_1, x_2 \geq 0.$

35. Use K.T. conditions to solve the problem :

Minimize $Z = 6x_1^2 + 5x_2^2$

subject to the constraints :

$x_1 + 5x_2 \geq 3$

$x_1, x_2 \geq 0.$

(2 × 15 = 30)