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B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2022

Fourth Semester

ABSTRACT ALGEBRA, LINEAR ALGEBRA, THEORY OF EQUATIONS, SPECIAL FUNCTIONS

(Complementary Course to Statistics)

(2013–2016 Admissions)

Time : Three Hours | Maximum Marks : 80

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Let $*$ be defined on Q by $a * b = ab.$ Is Q a group under $*$.
2. Give an example of a group of order n.
3. Define a skew field.
4. Define a Hermitian matrix.
5. Define an orthogonal matrix. What is the inverse of an orthogonal matrix.
6. Find the rank of $\begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}$.
7. What is a reciprocal equation ?
8. If α, β, γ are the roots of $x^3 + px^2 + qx + r = 0,$ find the value of $\sum \frac{1}{\beta\gamma}$ in terms of co-efficients.
9. Define the Gamma function.
10. Find the value of $\beta(2.5, 1.5)$.

(10 × 1 = 10)

Turn over

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Show that in a group G, the identity element and inverse element are unique.

12. Define Z_n and show that Z_n is a cyclic group.

13. Find the order of the cyclic subgroup of U_6 generated by $\cos\frac{2\pi}{3}+i\sin\frac{2\pi}{3}$.

14. Show that inverse of an orthogonal matrix is orthogonal.

15. Find the adjoint of the matrix $\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.

16. Find the rank of the matrix :

$$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{bmatrix}.$$

17. Solve the equation $x^4-2x^3-21x^2+22x+40=0$ whose roots are in A.P.

18. If α,β,γ are the roots of $x^3+qx+r=0$, find the value of $\sum\frac{\beta^2+\gamma^2}{\beta+\gamma}$.

19. Increase by 7 the roots of the equation $3x^4+3x^3+x^2-17x-19=0$.

20. Solve the equation $x^4-8x^3+19x^2-12x+2=0$ by removing its second term.

21. Show that $\beta(1,n)=\frac{1}{n}$.

22. Evaluate $\int_0^\infty x^3e^{-x^3}dx$.

(8 × 2 = 16)

Part C

*Answer any **six** questions.*
Each question carries 4 marks.

23. Let n be a positive integer and let $n\mathrm{Z} = \{nm \mid m \in \mathrm{Z}\}$. Show that $< n\mathrm{Z}, + >$ is a group.

24. Let G be a group and let $a, b \in \mathrm{G}$. Show that $(a*b)' = a'*b'$ if and only if $a*b = b*a$. Where a' denotes the inverse of a.

25. Compute the product in the rings given below :

    (a) $(20)(-8)$ in Z_{26}.

    (b) $(-3,5)(2,-4)$ in $\mathrm{Z}_4 \times \mathrm{Z}_{11}$.

26. If A is a Hermitian matrix, prove that iA is skew-Hermitian.

27. Deduce the following matrix to the normal form and hence find its rank :

$$\mathrm{A} = \begin{bmatrix} 0 & 1 & 2 & -2 \\ 4 & 0 & 2 & 6 \\ 2 & 1 & 3 & 1 \end{bmatrix}.$$

28. Prove that the set of vectors $(1,3,2), (1,-7,-8), (2,1,-1)$ of $\mathbb{R}^3$ is linearly dependent.

29. Check whether $w = \{(x, y, x, y) : x, y \in \mathrm{I}, \text{the set of integers}\}$ a subspace of $\mathbb{R}^4$.

30. If α, β, γ are the roots of the equation $x^3 + qx + r = 0$, find the equation whose roots are $(\beta - \gamma)^2, (\gamma - \alpha^2), (\alpha - \beta)^2$.

31. Prove that $\overline{|n} = (n-1)\overline{|}(n-1)$.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) Let G be a group and $a \in G$. Prove that $H = \{a^n, n \in Z\}$ is a subgroup of G and is the smallest subgroup of G that contains a.

    (b) Form the group Z_6 under addition and compute the subgroups , < 0 > and < 5 >.

33. (a) Find the eigen values and corresponding eigen vectors of the matrix :

$$\begin{bmatrix} 7 & -2 & -2 \\ -2 & 1 & 4 \\ -2 & 4 & 1 \end{bmatrix}.$$

    (b) Using Cayley-Hamilton theorem show that $A^3 - 6A^2 + 11\,A - 6I = 0$, where $A = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & 3 \end{bmatrix}$ and hence find A^{-1}.

34. (a) Solve the equation :

$$x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0.$$

    (b) Solve by Cardan's method :

$$x^3 - 18x - 35 = 0.$$

35. (a) Prove that $\beta(m,n) = \dfrac{\Gamma m \, \Gamma n}{\Gamma m+n}$.

    (b) Show that $\Gamma(n) = \int_0^1 \left(\log \dfrac{1}{x}\right)^{n-1} dx.$

(2 × 15 = 30)