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Reg. No.................................

Name....................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, SEPTEMBER 2024

Sixth Semester

Core Course – REAL ANALYSIS

(for B.Sc. Mathematics Model I, B.Sc. Mathematics Model II and B.Sc. Computer Applications)

(Prior to 2013 Admissions)

Time : Three Hours | Maximum Weight : 25

Part A

Answer **all** *the questions.*
Each bunch of four questions has weight 1.

I. 1 Show that the series $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}+\cdots$ is not convergent.

2 State Cauchy's general principle of convergence.

3 Define an alternating series.

4 Define a geometric series.

II. 5 State Raabe's test.

6 Define absolute convergence of a series.

7 Is the function $f(x)=\frac{x-|x|}{x}$ continuous at $x = 0$.

8 What is discontinuity of the first form ?

III. 9 Define uniform continuity of a function defined on an interval.

10 What is discontinuity of the second page ?

11 Define the upper sum $U(p,f)$ of a bounded real function defined on an interval.

12 Define the Riemann sum S (p, f).

Turn over

IV. 13 State the fundamental theorem of calculus.

14 State Cauchy's criterion for uniform convergence.

15 Is the sequence $\{x^n\}$ converges uniformly on [0, 1].

16 State Dirichlet's test for uniform convergence.

(4 × 1 = 4)

Part B

Answer any **five** *questions.*
Each question has weight 1.

17 Investigate the behaviour of the series whose n^{th} term is $\sin^{1/n}$.

18 Show that the series $1+\frac{1}{2!}+\frac{1}{3!}+\frac{1}{4!}+\cdots$ is convergent.

19 Test the convergence of the series whose n^{th} term is $\left\{\left(n^3+1\right)^{1/3}-n\right\}$.

20 Show that the function defined by $f(x)=\begin{cases}\frac{\sin 2x}{x}, \text{when } x \pm 0 \\ 1, \quad \text{when } x=0\end{cases}$ has a removable discontinuity at the origin.

21 Examine the continuity at x = 1 for the function $f(x)=\begin{cases}2x, \text{ if } 0 \le x \le 1 \\ 3, \quad \text{if } x=1 \\ 4x, \text{ if } 1 < x \le 2\end{cases}$.

22 Show that the constant function K is integrable and $\int_a^b \mathrm{K}\,dx=\mathrm{K}(b-a)$.

23 If f is continuous and non-negative on $[a, b]$, show that $\int_a^b f dx \geq 0$.

24 Show that the sequence $\{f_n\}$ where $f_n(x) = \frac{nx}{1+n^2x^2}$ is not uniformly convergent in any interval containing zero.

(5 × 1 = 5)

Part C

Answer any **four** *questions.*
Each question has weight 2.

25 Test the behaviour of the series $\frac{1^2.2^2}{1!} + \frac{2^2.3^2}{2!} + \frac{3^2.4^2}{3!} + \ldots$

26 Test the convergence of the series $\frac{x}{1} + \frac{1.x^3}{2.3} + \frac{1.3.x^5}{2.4.5} + \frac{1.3.5}{2.4.6}\frac{x^7}{7} + \ldots$

27 Show that the function f defined on R by $f(x) = \begin{cases} 1 \text{ if } x \text{ is rational} \\ 0 \text{ if } x \text{ is irrational} \end{cases}$ is discontinuous at every point.

28 Show that every continuous function is integrable.

29 If the functions f, f_1, f_2 where $f = f_1 \pm f_2$ are bounded and integrable on $[a, b]$, show that $\int_a^b f dx = \int_a^b f_1 dx \pm \int_a^b f_2 dx$.

30 State and prove Weierstrass μ test for uniform convergence.

(4 × 2 = 8)

Turn over

Part D

Answer any **two** *questions.*
Each question has weight 4.

31 State and prove D'Alembert's ratio test.

32 (a) Prove that a function which is continuous on a closed interval is also uniformly continuous on that interval.

(b) Show that the function $f(x) = \frac{1}{x}$ is not uniformly continuous on (0, 1).

33 Prove that a necessary and sufficient condition for the integrability of a bounded function *f* is that to every $\in > 0$, there corresponds $\delta > 0$ such that for every partition P of [*a*, *b*] with $\mu(p) < \delta, \mathrm{U}(p, f) - \mathrm{L}(p, f) < \in$.

(2 × 4 = 8)