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Reg. No.................................

Name....................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, SEPTEMBER 2024

Sixth Semester

Core Course—COMPLEX ANALYSIS

(For B.Sc. Mathematics Model I, Model II)

[Prior to 2013 Admissions]

Time : Three Hours Maximum Weight : 25

Part A

Answer **all** *questions.*

Each bunches of four questions carries a weight of 1.

I. 1 Find the domain of the function $f(z) = \text{Arg}\left(\frac{1}{z}\right)$

2 Define limit of a function.

3 Find $f'(z)$, when $f(z) = \left(1-4z^2\right)^3$.

4 Define an analytic function.

II. 5 Give an example of an entire function.

6 What is the period of e^z.

7 Define sinh$z$ cosh$z$.

8 Evaluate $\int_1^2 \left(\frac{1}{t}-i\right)^2 dt$.

Turn over

III. 9 Give an example of a simple closed contour.

10 If C is any simple closed contour, find $\int_C \exp\left(z^3\right) dz$.

11 Define a simply connected domain.

12 Define absolute convergence of a series of complex numbers.

IV. 13 What are the isolated singularities of $\dfrac{1}{\sin\left(\pi/z\right)}$.

14 Define residue of a function *f*.

15 Define removable singularity.

16 What type of singularity the function $e^{1/z}$ have.

(4 × 1 = 4)

Part B

Answer any **five** *questions.*

Each question carries a weight of 1.

17 Show that $f'\left(z\right)$ does not exist at any point if $f\left(z\right) = \bar{z}$.

18 Write the singularities of $f\left(z\right) = \dfrac{z^3 + 4}{\left(z^2 - 3\right)\left(z^2 + 1\right)}$.

19 If $f\left(z\right) = \cosh x \cos y + i \sinh x \sin y$. Find u_x, u_y, v_x, v_y.

20 Evaluate $\int_C \dfrac{z+2}{z}$, where C is the circle $z = 2e^{i\theta}, 0 \le \theta \le 2\pi$.

21 Use Cauchy-Goursat theorem to evaluate $\int_C \dfrac{1}{z^2 + 2z + 2} dz$ where C is the unit circle $\left|z\right| = 1$.

22 State Laurent's theorem.

23 Find the residue at $z = 0$ of the function $z \cos\left(\frac{1}{z}\right)$.

24 Find the order of the pole and residue of the function $\frac{1 - \cosh z}{z^3}$ at the pole.

(5 × 1 = 5)

Part C

Answer any **four** *questions.*

Each question carries a weight of 2.

25 Show that $f'(z)$ and $f''(z)$ exist every where and find $f''(z)$ for the function $f(z) = iz + 2$.

26 Show that if $f'(z) = 0$ everywhere in a domain D, then $f(z)$ must be constant throughout D.

27 Find the value of the integral of $g(z)$ around the circle $(z - i) = 2$ in the positive sense when $g(z) = \frac{1}{z^2 + 4}$.

28 State and prove Cauchy's inequality.

29 Find the Laurent series that represents the function $f(z) = z^2 \sin\left(\frac{1}{z^2}\right)$ in the domain $0 < |z| < \infty$.

30 Use Cauchy's residue theorem to evaluate the integral of $\frac{\exp(-z)}{z^2}$ around the circle $|z| = 3$ in the positive sense.

(4 × 2 = 8)

Part D

Answer any **two** *questions.*

Each question carries a weight of 4.

31 (a) State and prove fundamental theorem of algebra.

(b) State maximum modules principle.

32 Evaluate $\int_0^{\infty} \frac{x^2}{x^6+1}\, dx$.

33 (a) Show that when $0 < |z| < 4$ $\frac{1}{4z - z^2} = \frac{1}{4z} + \sum_{n=0}^{\infty} \frac{z^4}{4^{n+2}}$.

(b) State and prove Taylor's theorem.

(2 × 4 = 8)