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Reg. No.................................

Name...................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, SEPTEMBER 2024

Sixth Semester

Core Course—LINEAR ALGEBRA AND METRIC SPACES

(for B.Sc. Mathematics Model I and Model II)

(Prior to 2013 Admissions)

Time : Three Hours | Maximum Weight : 25

Part A

Answer **all** *questions.*
Each bunch of 4 *four questions carries a weight of* 1.

I. 1 Is the real line R, a vector space over C, the complex field.

2 Define linear independence of a set of vectors.

3 What is the dimension of a vector space ?

4 Give an example of an infinite dimension of a vector space.

II. 5 Define a basis for a vector space.

6 Which vector is invariant under every linear transformation ?

7 Define Kernel of a linear transformation.

8 Define zero mapping and identify mapping.

III. 9 Which we say two vector spaces are isomorphic ?

10 What is the usual metric on R ?

11 Define an open sphere in a metric space.

12 Define interior of a set.

IV. 13 How will you define a closed set in a metric space ?

14 Define convergence of a sequence.

15 Define a complete metric space.

16 State Baire's theorem.

$(4 \times 1 = 4)$

Turn over

Part B

Answer any **five** *questions.*
Each question carries a weight of 1.

17 Show that the vectors $(1,0,0),(0,1,0)$ and $(0,0,1)$ form a spanning set of R^3.

18 Show that intersection of two subspaces of a vector space is also a subspace of the vector space.

19 Show that the mapping $T : R^3 \to R$ defined by $T(x,y,z) = 2x - 3y + 4z$ is linear.

20 Let $F : V \to U$ be a linear transformation. Show that Kernel of F is a subspace of V.

21 Let X be a non-empty set, and let d be a real function ordered pairs of elements of X which satisfies the conditions $d(x,y) = 0 \Leftrightarrow x = y$ and $d(x,y) \leq d(x,z) + d(y,z)$. Show that d is a metric on X.

22 Show that in any metric space X, the empty set ϕ and the full space X are open sets.

23 Define the Canter set.

24 Let X be an arbitrary non-empty set define d by $d(x,y) = \begin{cases} 0 \text{ if } x = y \\ 1 \text{ if } x \pm y \end{cases}$. Show that any mapping defined on X into any other metric space is continuous.

(5 × 1 = 5)

Part C

Answer any **four** *questions.*
Each question carries a weight of 2.

25 Show that a set of vectors which contains at least one zero vector is linearly dependent.

26 Find a basis and dimension of the subspace W of $V = \mu_{22}$ spanned by :

$$A = \begin{bmatrix} 1 & -5 \\ -4 & 2 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ -1 & 5 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & -4 \\ -5 & 7 \end{bmatrix}, D = \begin{bmatrix} 1 & -7 \\ -5 & 1 \end{bmatrix}.$$

27 Let $T : R^2 \to R^2$ be such that $T(1,1) = (3,1)$ and $T(0,1) = (-2,0)$. Find T (a, b) for any $a, b \in R$.

28 Test whether $F : R^2 \to R^3$ defined by $F(x, y) = (x+3, 2y, x+y)$ linear.

29 Let X be a metric space. Show that a subset G of X is open if and only if it is a union of open spheres.

30 Let X be a metric space if $\{x_n\}$ and $\{y_n\}$ are sequences in X such that $x_n \to x$ and $y_n \to y$, show that $d(x_n, y_n) \to d(x, y)$.

(4 × 2 = 8)

Part D

Answer any **two** *questions.*
Each question carries a weight of 4.

31 (a) Let V be the vector space of ordered pairs of complex numbers over the real field R. Show that the set $S = \{(1,0), (i,0), (0,1), (0,i)\}$ is a basis of V.

(b) Prove that the set of all continuous functions on [0, 1] is a vector space and the set of all polynomials in [0,1] is a subspace of it.

32 (a) Let $T : V \to V$ be a linear map and $X_1, X_2, \ldots\ldots, X_n \in V$. If T (X_1), T (X_2), ... T (X_n) are linearly independent vectors of V, then prove that X_1, X_2, ...,X_n are also linearly independent.

(b) Find the matrix of the linear transformation T on R^3 defined by T $(x, y, z) = (2y + z, x - 4y, 3x)$ with respect to the ordered basis B = {(1, 1, 1), (1, 1, 0), (1, 0,0)}.

33 Let X and Y be metric spaces and f be a mapping of X into Y. Prove that f is continuous if and only if f^{-1}(G) is open in X whenever G is open in Y.

(2 × 4 = 8)