

E 6165



Reg.	No
Nam	16

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, SEPTEMBER 2024

Sixth Semester

Choice Based Course—TOPOLOGY

(for B.Sc. Mathematics Model I)

(Prior to 2013 Admissions)

Time: Three Hours Maximum Weight: 25

Part A

Answer **all** questions.

Each bunch of four questions has weight 1.

- I. 1 Define topology on a set X.
 - 2 Define finite complement topology.
 - 3 Define lower limit topology on R.
 - 4 What are projection mappings?
- II. 5 Define subspace topology.
 - 6 Is the set $\{x \times y / x \ge 0 \text{ and } y \ge 0\}$ closed in the plane \mathbb{R}^2 .
 - 7 Define interior of a set.
 - 8 Define limit point of a set.
- III. 9 Give an example of a Hausdorff space.
 - 10 Is the set Q of rationals connected.
 - 11 Define a linear continuum.
 - 12 What is a path connected space?
- IV. 13 Define a compact space.
 - 14 Is (0, 1) compact subset of R.
 - 15 Is a finite set compact.
 - 16 Is the real line R compact.

 $(4 \times 1 = 4)$

1/3 Turn over





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Part B

Answer any **five** questions. Each question has weight 1.

- If $\{\mathfrak{I}_{\alpha}\}$ is a collection of topologies on X, show that $\cap \mathfrak{I}_{\alpha}$ is a topology on X. Is $\cap \mathfrak{I}_{\alpha}$ a topology on X.
- 18 Let Y be a subspace of X. Show that if \cup is open in Y and Y is open in X, then \cup is open in X.
- 19 Show that every finite point set in a Hausdorff space X is closed.
- 20 Let R denote the set of real numbers in the usual topology, and let R_l denote the same set in the lower limit topology. Let $f: R \to R_l$ be the identity function f(x) = x. Is f a continuous function. Justify your answer.
- 21 State intermediate value theorem.
- 22 What is a totally disconnected space. Give an example.
- 23 Show that every closed subset of a compact space is compact.
- 24 State the tube lemma.

 $(5 \times 1 = 5)$

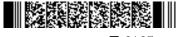
Part C

Answer any **four** questions. Each question has weight 2.

- 25 If \mathcal{B} is a basis for the topology of X and \mathcal{C} is a basis for the topology of Y, then show that the collection $D = \{B \times C / \mathcal{B} \in B \text{ and } C \in \mathcal{C}\}$ is a basis for the topology of X \times Y.
- 26 Let Y be a subspace of X, show that a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y.
- 27 If $f: X \to Y$ and $g: Y \to Z$ are continuous, then show that $g \circ f: X \to Z$ is continuous.
- 28 Find a function $f: \mathbb{R} \to \mathbb{R}$ that is continuous at precisely one point.
- 29 Show that union of a collection of connected sets that have a point in common is connected.
- 30 Show that the image of a compact space under a continuous map is compact.

 $(4 \times 2 = 8)$





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Part D

Answer any **two** questions. Each question has weight 4.

- 31 Let $f: A \to X \times Y$ be given by $f(a) = (f_1(a), f_2(a))$. Prove that f is continuous if and only if the functions $f_1: A \to X$ and $f_2: A \to Y$ are continuous.
- 32 (a) Show that the image of a connected space under a continuous map is connected.
 - (b) Let A be a connected subset of X. If $\,A \subset B \subset \overline{A}$, show that B is also connected.
- 33 Show that product of finitely many compact spaces is compact.

 $(2 \times 4 = 8)$

