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Reg. No.................................

Name....................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, SEPTEMBER 2024

Sixth Semester

Choice Based Course—TOPOLOGY

(for B.Sc. Mathematics Model I)

(Prior to 2013 Admissions)

Time : Three Hours　　　　Maximum Weight : 25

Part A

Answer **all** *questions.*
Each bunch of four questions has weight 1.

I. 1 Define topology on a set X.

2 Define finite complement topology.

3 Define lower limit topology on R.

4 What are projection mappings ?

II. 5 Define subspace topology.

6 Is the set $\{x \times y / x \geq 0 \text{ and } y \geq 0\}$ closed in the plane R^2.

7 Define interior of a set.

8 Define limit point of a set.

III. 9 Give an example of a Hausdorff space.

10 Is the set Q of rationals connected.

11 Define a linear continuum.

12 What is a path connected space ?

IV. 13 Define a compact space.

14 Is (0, 1) compact subset of R.

15 Is a finite set compact.

16 Is the real line R compact.

(4 × 1 = 4)

Part B

Answer any **five** *questions.*
Each question has weight 1.

17 If $\{\Im_\alpha\}$ is a collection of topologies on X, show that $\cap\Im_\alpha$ is a topology on X. Is $\cap\Im_\alpha$ a topology on X.

18 Let Y be a subspace of X. Show that if $\cup$ is open in Y and Y is open in X, then $\cup$ is open in X.

19 Show that every finite point set in a Hausdorff space X is closed.

20 Let R denote the set of real numbers in the usual topology, and let R_l denote the same set in the lower limit topology. Let $f : R \to R_l$ be the identity function $f(x) = x$. Is f a continuous function. Justify your answer.

21 State intermediate value theorem.

22 What is a totally disconnected space. Give an example.

23 Show that every closed subset of a compact space is compact.

24 State the tube lemma.

(5 × 1 = 5)

Part C

Answer any **four** *questions.*
Each question has weight 2.

25 If $\mathscr{B}$ is a basis for the topology of X and $\mathscr{C}$ is a basis for the topology of Y, then show that the collection $D = \{B \times C / \mathscr{B} \in B \text{ and } C \in \mathscr{C}\}$ is a basis for the topology of X × Y.

26 Let Y be a subspace of X, show that a set A is closed in Y if and only if it equals the intersection of a closed set of X with Y.

27 If $f : X \to Y$ and $g : Y \to Z$ are continuous, then show that $g \circ f : X \to Z$ is continuous.

28 Find a function $f : R \to R$ that is continuous at precisely one point.

29 Show that union of a collection of connected sets that have a point in common is connected.

30 Show that the image of a compact space under a continuous map is compact.

(4 × 2 = 8)

Part D

Answer any **two** *questions.*
Each question has weight 4.

31 Let $f : \mathrm{A} \to \mathrm{X} \times \mathrm{Y}$ be given by $f(a) = (f_1(a), f_2(a))$. Prove that f is continuous if and only if the functions $f_1 : \mathrm{A} \to \mathrm{X}$ and $f_2 : \mathrm{A} \to \mathrm{Y}$ are continuous.

32 (a) Show that the image of a connected space under a continuous map is connected.

(b) Let A be a connected subset of X. If $\mathrm{A} \subset \mathrm{B} \subset \bar{\mathrm{A}}$, show that B is also connected.

33 Show that product of finitely many compact spaces is compact.

(2 × 4 = 8)