INTEGRATED PG DEGREE EXAMINATION, DECEMBER 2023

Third Semester

INTEGRATED MSC BASIC SCIENCE-CHEMISTRY

Complementary - ICH3CM04 - PARTIAL DIFFERENTIAL EQUATIONS, ABSTRACT ALGEBRA AND ANALYTIC GEOMETRY

2020 ADMISSION ONWARDS

0E311DAA

Time: 3 Hours   Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Find the focus and directrix of the parabola $y = 4x^2$.

2. Find the equation of the hyperbola with foci $(0, \pm 4)$, vertices $(0, \pm 5)$ in the standard form.

3. Find the directrix of the parabola $r = \frac{25}{10 + 10\,cos\,\theta}$.

4. Is multiplication on the set of all Reral numbers a binary operation . Justify your answer.

5. Define finite group. Give an example.

6. Give an example of a cyclic subgroup of Z ,the set of integers under addition as composition.

7. Determine whether f(x) = x^2 is a permutation on R.

8. Write the geometrical interpretation of Newton Raphson method.

9. Find the order and degree of the partial differential equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} + \left(\frac{\partial z}{\partial y}\right)^3 = 0$.

10. Find the integral curves of $\frac{dx}{z} = \frac{dy}{0} = \frac{dz}{-x}$.
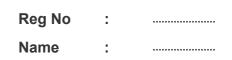
(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Find the centre, eccentricity, foci, vertices and directrices of the ellipse $\frac{(x+1)^2}{25} + \frac{(y-3)^2}{16} = 1$.

12. Find the polar equation of the circle $(x - 3)^2 + (y + 1)^2 = 4$.

13. Show that if H and K are two Subgroups of a group G ,then $H \cap K$ is a subgroup of G.

14. Prove that subgroup of a cyclic group is cyclic.

15. Find a real root of the equation $x^3 - x^2 - 1 = 0$ correct to 3 decimal places using Bisection method.

16. Find a real root of the equation $x^3 - 2x - 5 = 0$ using the Method of False position correct to 3 decimal places.

17. Form a partial differential equation by eliminating arbitrary constants a and b from
$z = (x - a)^2 + (y - b)^2$.

18. Obtain a partial differentail equation by eliminating the arbitrary function from $z = f(xy) + g(\frac{x}{y})$.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. i) Replace the polar equation with equivalent cartesian equation. a) $r sin\theta = 0$, b) $r\, cos\theta + r\, sin\theta = 1$,
    ii) Replace the cartesian equation with equivalent polar equations , a) $xy = 2$ b) $(x - 5)^2 + y^2 = 25$.

20. Let ϕ be a homomorphism of a group G into a group G'.
    a) If H is a subgroup of G then prove that $\phi(H)$ is a subgroup of G'.
    b) If K' is a subgroup of G' then prove that $\phi^{-1}(K')$ is a subgroup of G.

21. Find a real root of the equation $x^3 + x^2 - 1 = 0$ on the interval [0,1] with an accuracy of 10^{-4} using the Method of Iteration.

22. Solve $(t + y + z)\frac{\partial t}{\partial x} + (t + z + x)\frac{\partial t}{\partial y} + (t + x + y)\frac{\partial t}{\partial z} = x + y + z.$

(2×5=10 weightage)