QP CODE: 24803818

24803818

Reg No : .....

Name : .....

INTEGRATED MSC DEGREE EXAMINATION, JULY 2024

Fourth Semester

INTEGRATED MSC BASIC SCIENCE-CHEMISTRY

COMPLEMENTARY - ICH4CM04 - FOURIER SERIES, LAPLACE TRANSFORMATIONS, VECTOR FUNCTIONS AND COMPLEX ANALYSIS

2020 Admission Onwards

6217CC28

Time: 3 Hours | Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define Fourier series of a function $f(x)$ with period $2L$.
2. Define Fourier Cosine series.
3. Find $L(cos\ 4t)$
4. Write $L(\frac{f(t)}{t})$ in terms of an Integral.
5. Write the Laplace Transform of the second derivative of f(t).
6. Prove that $div(grad\ f) = \nabla^2 f$.
7. State Divegence theorem.
8. Prove that $|z_1 + z_2|^2 + |z_1 - z_2|^2 = 2|z_1|^2 + 2|z_2|^2$.
9. Define Analytic function with example.
10. State Cauchy Integral Formula.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Find the Fourier series expansion of $f(x) = \begin{cases} 0 & when \ -\pi < x < 0 \\ 1 & when \ \ 0 < x < \pi \end{cases}$ and $f(x + 2\pi) = f(x)$.

12. Find the Fourier series of the function $f(x) = x + \pi \, , if \ -\pi < x < \pi$ and $f(x + 2\pi) = f(x)$.

13. Evaluate $L(sin\ at\ sin\ bt)$

14. Evaluate $L^{-1}(\frac{6s-4}{s^2-4s+20})$

15. Find the Directional Derivative of $f(x, y, z) = ln(x^2 + y^2)$ at $P(3, 0)$in the direction of $\hat{i} - \hat{j}$.

16. Evaluate the line integral of $f(x, y, z) = 3x^2 - 2y + z$ over line segment C joining the origin to the point $(2, 2, 2)$.

17. Find and all the cube roots of $8i$.

18. Show that $\int_c \frac{dz}{z} = 2\pi i$ where C is the unit circle in the counter clock wise sense.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Find the half range sine expansion of $f(x) = e^x \ , \ 0 < x < 2$.

20. Solve $y'' \ - \ 3y' \ + \ 2y = 4e^{2t}. \ y(0) = -3, \ y'(0) = 5.$

21. Verify Green's theorem for the field $\vec{F} = xy \ \hat{i} + x^2 \ \hat{j}$ and the region R bounded by the parabola $y = x^2$ and the line $y = x$.

22. Show that $u = 4xy \ - \ x^3 \ + \ 3xy^2$ is a Harmonic function. Find its harmonic conjugate and hence the most general analytic function $f(z)$ with u as real component.

(2×5=10 weightage)