24803740

QP CODE: 24803740

Reg No : ....

Name : ....

INTEGRATED MSC DEGREE EXAMINATION, JUNE 2024

Second Semester

COMPLEMENTARY - ICSC2CM5 - MATHEMATICS – II-LINEAR ALGEBRA

INTEGRATED MSC COMPUTER SCIENCE-ARTIFICIAL INTELLIGENCE AND MACHINE LEARNING & INTEGRATED MSC COMPUTER SCIENCE- DATA SCIENCE

2020 Admission Onwards

CC3C14AF

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Consider the statement that 0 is an additive identity for $\mathbb{F}^n$: x+0=x for all x $\in \mathbb{F}^n$. Is the 0 in the equation the number 0 or the list 0? Justify your answer.
2. Why the empty set is not considered as a vector space?
3. Define a surjective function on linear map.
4. What can we say about the solutions of the following systems of linear equations in terms of number of equations and variables? a)homogeneous b) inhomogeneous.
5. Is $\mathbb{R}^2 \times \mathbb{R}^3$ isomorphic to $\mathbb{R}^5$? Justify your answer.
6. Define an affine subset of a vector space.
7. What is the difference between the upper triangular matrix and a diagonal matrix?
8. When we say that an operator is diagonalizable?
9. What is the condition for two vectors are orthogonal? Check whether the vectors $(1, 0, -1)$,$(1, \sqrt{2}, 1)$ are orthogonal.
10. Define square root of linear operator.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. For the given subsets of $\mathbb{F}^3$, determine whether they are subspaces of $\mathbb{F}^3$.
a) $\{(x_1, x_2, x_3) \in \mathbb{F}^3 : x_1 = 5x_3\}$.
b) $\{(x_1, x_2, x_3) \in \mathbb{F}^3 : x_1 + 2x_2 + 3x_3 = 4\}$.

12. Prove that every linearly independent list of vectors in a finite dimensional vector space V with length dim V is a basis of V.

13. Explain the algebraic properties of products of linear maps.

14. a) Define the matrix of a linear map $\mathcal{M}(\mathcal{T})$.
b) Let $\mathcal{T} \in L(\mathbb{F}^2, \mathbb{F}^3)$ be defined by $\mathcal{T}(x, y) = (x + 3y, 2x + 5y, 7x + 9y)$. Find the matrix of $\mathcal{T}$ with respect to the standard bases of $\mathbb{F}^2$ and $\mathbb{F}^3$.

15. Find the eigenvalues and eigenvectors of the matrix $\begin{bmatrix} 2 & 6 \\ 1 & 3 \end{bmatrix}$.

16. Let $\mathcal{T} \in \mathcal{L}(\mathcal{V})$. If $\mathcal{T}/(null\ \mathcal{T})$ is injective, then prove that $(null\ \mathcal{T}) \cap (range\ \mathcal{T}) = \{0\}$.

17. Suppose $T$is an operator on $\mathbb{F}^2$ whose matrix with respect to the standard basis is $\begin{pmatrix} 2 & -3 \\ 3 & 2 \end{pmatrix}$.Show that T is not self adjoint and T is normal.

18. Give an example of $T \in \mathcal{L}(\mathbf{C}^2)$ such that θT is the only eigenvalue of and the singular values of T are $5, 0$.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a) State and prove the Linear Dependence Lemma.
b) Prove that in a finite-dimensional vector space, the length of every linearly independent list of vectors is less than or equal to the length of every spanning list of vectors.

20. a) Suppose V is finite dimensional and $\mathcal{T} \in \mathcal{L}(V)$. Then prove that the following are equivalent:
i) $\mathcal{T}$ is invertible
ii) $\mathcal{T}$ is injective
iii) $\mathcal{T}$ is surjective

b) Show that for each polynomial $q \in \mathcal{P}(\mathbb{R})$, there exists a polynomial $p \in \mathcal{P}(\mathbb{R})$, with $((x^2+5x+7)p)'' = q$.

21. State and prove the theorem on the eigenvalues of the operators on complex vector spaces.

22. Give an example of an orthonormal basis of $\mathcal{P}_2(\mathbf{R})$ where the inner product is given by $< p, q >= \int_{-1}^{1} p(x)q(x)dx$.

(2×5=10 weightage)