QP CODE: 24801183

24801183

Reg No : .....

Name : .....

INTEGRATED MSC DEGREE EXAMINATION, FEBRUARY 2024

First Semester

INTEGRATED MSC BASIC SCIENCE-PHYSICS

Complementary - IPH1CM05 - MATHEMATICS - I DIFFERENTIAL CALCULUS, MATRICES AND TRIGONOMETRY

2021 Admission Onwards

E737A702

Time: 3 Hours | Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Find critical points for the function $y = x^2 - 6x + 7$.
2. Find the critical points for the function $f(x) = x^3 - 12x - 5$.
3. Find $\frac{\partial^2 f}{\partial x^2}$ and $\frac{\partial^2 f}{\partial y^2}$ if $f(x, y) = x + y + xy$.
4. Find $\frac{dw}{dt}$ if $w = x^2 + y^2, x = \cos t, y = \sin t$.
5. Use implicit differentiation, find $\frac{dy}{dx}$ if $y^2 - x^2 - \sin(xy) = 0$.
6. Define augmented matix.
7. Define characteristic vector.
8. If $x = \cos\theta + i\sin\theta$, find $x^6 + \frac{1}{x^6}$ and $x^6 - \frac{1}{x^6}$.
9. Prove that $\cosh 2x = \cosh^2 x + \sinh^2 x$.
10. Separate into real and imaginary parts the expression $\cosh(x + iy)$.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Verify mean value theorem for the function $f(x) = x^2 + 2x - 1$ on $[0, 1]$.
12. Find $\lim_{x\to\infty}(\frac{1}{\sin x} - \frac{1}{x})$.

13. a. Find f_x and f_y as a function if $f(x,y) = \frac{2y}{y+\cos x}$.
    b. If $u = a\tan^{-1}(\frac{y}{x})$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$.

14. Evaluate $\frac{dw}{dt}$ at $t = 1$ if $w = 2ye^x - ln\ z,\ x = \ln(t^2+1),\ y = \tan^{-1} t,\ z = e^t$.

15. If $A = \begin{bmatrix} 2 & 1 & 3 \\ 6 & 7 & 8 \\ 5 & 4 & 2 \end{bmatrix}$, then verify $(A')' = A$.

16. Verify Cayley Hamilton theorem for the matrix $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

17. If $\tan\theta = \frac{1}{2}$, find the value of $\tan 5\theta$.

18. Sum the series $\cos\theta + \frac{\cos\theta}{1!}\cos 2\theta + \frac{cos^2\theta}{2!}\cos 3\theta + \frac{cos^3\theta}{3!}\cos 4\theta + \ldots$ to ∞.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a. Find the domain and range of the function $f(x,y) = \sqrt{9 - x^2 - y^2}$.
    b. Find an equation for the level curve of the function $f(x,y) = \sqrt{x-y} - lnz$.
    c. Sketch the domain for the function $f(x,y) = ln(x^2 + y^2 - 4)$.

20. Solve using Cramer's rule
$$\begin{aligned} 2x_1 + x_2 + 5x_3 + x_4 &= 5 \\ x_1 + x_2 - 3x_3 - 4x_4 &= -1 \\ 3x_1 + 6x_2 - 2x_3 + x_4 &= 8 \\ 2x_1 + 2x_2 + 2x_3 - 3x_4 &= 2 \end{aligned}$$

21. Find the eigen values of $\begin{bmatrix} 1 & 2 & 2 \\ 0 & 2 & 1 \\ -1 & 2 & 2 \end{bmatrix}$.

22. Sum the following series
    a. $1 + a\cos\theta + a^2\cos 2\theta + a^3\cos 3\theta + \ldots$, where $|a| < 1$.
    b. $\cos\theta + \frac{1}{2}\cos 2\theta + \frac{1.3}{2.4}\cos 3\theta + \frac{1.3.5}{2.4.6}\cos 4\theta + \ldots$ to ∞.

(2×5=10 weightage)