QP CODE: 23800345

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Reg No : ....

Name : ....

INTEGRATED PG DEGREE EXAMINATION, DECEMBER 2023

Third Semester

INTEGRATED MSC BASIC SCIENCE-PHYSICS

Complementary - IPH3CM04 - MATHEMATICS - III DIFFERENTIAL EQUATIONS AND VECTOR CALCULUS

2020 ADMISSION ONWARDS

4C18A358

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Verify that the function $x^2 + y^2 = c$ is a solution of the differential equation $y\frac{dy}{dx} + x = 0$.
2. Solve the initial value problem $x\frac{dy}{dx} + y = 0, y(2) = -2$.
3. Examine whether the differential equation $(2x + e^y)dx + xe^y dy = 0$ is exact or not.
4. Solve $\frac{dx}{z} = \frac{dy}{0} = \frac{dz}{-x}$.
5. Form a partial differential equation by eliminating the arbitrary function f from $z = f(\frac{xy}{z})$.
6. State the dot product rule of vector valued functions
7. Find the length of the catenary r(t)= t **i** +Cosht **j**, from t=0 to t=1
8. Define circle of curvature at a point on a plane curve.
9. Define the line integral of f over a curve C.
10. Define curl of a vector field $\mathbf{F}$.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Solve the differential equation $(3x^2y + e^y)dx + (x^3 + xe^y - 2y)dy = 0$.

12. Solve the differential equation $(x^2 - 2x + 2y^2)dx + 2xydy = 0$ by finding the integrating factor.

13. Write a note about the direction cosines of the tangent to a curve in parametric form.

14. How can we solve Lagranges equation?

15. Define an arc of a curve. What is the vector equation of a straight line?

16. Give three important properties of directional derivative $D_u f$.

17. Find the line integral of $F = 3y\,\mathbf{i} + 2x\,\mathbf{j} + 4z\,\mathbf{k}$, from $(0,0,0)$ to $(1,1,1)$ along the curve $r(t) = t\,\mathbf{i} + t^2\,\mathbf{j} + t^4\,\mathbf{k}; 0 \le t \le 1.$

18. A fluid's velocity field is $F = -4xy\mathbf{i} + 8y\mathbf{j} + 2\mathbf{k}$. Find the flow along the curve $r(t) = t\mathbf{i} + t^2\mathbf{j} + \mathbf{k}; 0 \le t \le 2.$

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a) Solve $\frac{dy}{dx} + xsin2y = x^3cos^2y.$
b) Solve $x^3\frac{dy}{dx} + 3x^2y = cosx.$

20.
  1. Form a partial differential equation by eliminating a and b from $\frac{x^2+y^2}{a^2} + \frac{z^2}{b^2} = 1.$.
  2. Find the partial differential equation from $2z = (ax+y)^2 + b$ by eliminating the arbitrary constants.

21. (a) Find the derivative of h(x,y,z) = Cos (xy) + e^{yz} + In zx at the point P_0 (1,0,1/2) in the direction of the vector **i** +2**j** +2**k**
(b) In what direction does h change most rapidly at P_0 and what are the rates of change in the directions

22. State normal form and tangential form of Green's theorem. Verify both forms of Green's theorem for the field$F(x,y) = (x-y)\mathbf{i} + x\mathbf{j}$ and the region bounded by the unit circle $C: r(t) = \cos t\mathbf{i} + \sin t\mathbf{j}; 0 \le t \le 2\pi.$

(2×5=10 weightage)