QP CODE: 24803833

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Reg No : .....

Name : .....

INTEGRATED MSC DEGREE EXAMINATION, JULY 2024

Fourth Semester

INTEGRATED MSC BASIC SCIENCE-PHYSICS

COMPLEMENTARY - IPH4CM04 - SPECIA L FUNCTIONS, LA PLACE TRANSFORMS AND COMPLEX ANALYSIS

2021 Admission Onwards

600BE2A0

Time: 3 Hours | Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. How do you solve a differential equation using power series method?
2. Explain Bessel's equation
3. Find the Laplace Transform of the function $f(t) = e^{at}$.
4. Write $L[t^3 f(t)]$ using a derivative of $L[f(t)]$.
5. Find the inverse laplace transform of $\frac{1}{s^2+1}$.
6. Find the real and imaginary parts of the complex function $f(z) = z^2 + 3z$ and calculate the value of f at $z = 1 + 3i$.
7. Show that $exp(z + \pi i) = -e^z$.
8. Evaluate i^{2i}.
9. Define parametric representation of an arc. Also find the parametric representation $z = z(t)$ of the line segment with end points $z = 0$ and $z = 1 + 2i$.
10. (a) What is the formula to find $f'(z_0)$ where $f(z)$ is an analytic function in a domain D over a simple closed contour C in D which encloses z_0 oriented in the counter clockwise direction?
    (b) Write the formula to find $f'(z_0)$ where $f(z)$ is an analytic function in a domain D bounded by simple closed contours C_1 and C_2 and z_0 is a point in that domain.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Derive $P_3(x)$ from $P_n(x)$ of Legendre polynomial
12. Solve $y'' = y$
13. Find the Laplace Transform of the function $f(t) = e^{a+bt}$.
14. Find the Laplace Transform of $\sinh t \cos t$.
15. If u and v are harmonic functions conjugate to each other in some domain, then prove that u and v must be constant there.
16. Show that for any two complex numbers z_1 and z_2, $2 \sin z_1 \cos z_2 = \sin(z_1 + z_2) + \sin(z_1 - z_2)$.
17. Evaluate $\int_c \sin^2 z \, dz$ where C is any curve varies from $-\pi i$ along $|z| = \pi$ to πi in the right half plane.
18. Evaluate $\oint_c \frac{1}{z^4-1} \mathrm{d}z$ where C is the circle $|z + i| = 1$oriented in the counter clockwise sense.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Give Legendre's equation and find its general solution
20. Solve the following Laguerre's equation, where n is an integer: $ty'' + (1 - t)y' + ny = 0$.
21. (a). Find and plot $\sqrt[3]{-7 + 4i}$.
    (b). Sketch the regions $(i) Re(z) \geq 1 \; and \; Im(z) \leq 1 \; (ii) \mid 2z + 3 \mid > 4$. In each case describe whether they are open or closed and which of them are domains.
22. (a) Explain the principle of deformation.
    (b)Using the principle of deformation and the fact that $\frac{2z-1}{z^2-z} = \frac{1}{z} + \frac{1}{z-1}$, show that $\int_c \frac{2z-1}{z^2-z} \mathrm{d}z = 4\pi i$ where C is any closed contour enclosing $z = 0$ and $z = 1$ in the counter clockwise direction.

(2×5=10 weightage)