QP CODE: 24801172

24801172

Reg No : .....

Name : .....

INTEGRATED MSC DEGREE EXAMINATION, FEBRUARY 2024

First Semester

INTEGRATED MSC BASIC SCIENCE-STATISTICS

Complementary - IST1CM05 - MATHEMATICS I-LOGIC, SET THEORY AND DIFFERENTIAL CALCULUS

2020 Admission Onwards

C010D4AD

Time: 3 Hours | Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. What is the truth value of $\forall x P(x)$, where P(x) is the statement "$x^2 < 10$" and the domain consists of the positive integers not exceeding 4?
2. Define modus ponens.
3. a) Define the cardinality of a set S.
   b) Let S be the set of letters in the English alphabet. Then |S|= ?
4. Define right-hand limit and left-hand limit.
5. Find the derivative of $f(x) = \sqrt{x}$ for x>0. Also find $f^1(4)$.
6. Find the derivative of $y = \frac{t^2-1}{t^3+1}$.
7. Prove the reciprocal rule for logarithms.
8. Find the intervals on which the function $f(x) = ax^2 + bx + c$, $a \neq 0$ is increasing and decreasing.
9. Draw a branch diagram of the chain rule formula for differentiable function $w = f(x, y, z)$ when $x = x(t),\ y = y(t)\ and\ z = z(t)$ are both differentiable functions of t.
10. Draw a branch diagram for the chain rule for $f(x, y, z)$ where $x = g(r, s)\ , y = h(r, s)\ and\ z = k(r, s)$.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. What are the contrapositive, the converse, and the inverse of the conditional statement "The home team wins whenever it is raining?"

12. Prove that if n is an integer and 3n + 2 is odd, then n is odd.

13. Estimate the value of $\lim_{x\to 0} \frac{\sqrt{x^2+100}-10}{x^2}$.

14. a) Define the slope of the curve.
    b) Find the slope of the circle $x^2 + y^2 = 25$ at the point (3,-4).

15. Sketch the graph of $y = \frac{6}{x^2+2}$, $-1 < x < 1$ and determine whether the function has any absolute extreme values on its domain.

16. Determine all critical points of $g(x) = (x-1)^2(x-3)^2$.

17. Find $\frac{\partial^5 f}{\partial x^2 \partial y^3}$ for $f(x,y) = x\, e^{\frac{y^2}{2}}$.

18. Extend the formula for implicit differentiation for 3 variables.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a) Define bijective function.
    b) Determine whether the function f (x) = x^2 from the set of integers to the set of integers is one-to-one.
    c) Let f be the function from {a, b, c, d} to {1, 2, 3} defined by f (a) = 3, f (b) = 2, f (c) = 1, and f (d) = 3. Is f an onto function?

20. Find derivatives of the following functions
    a) $y = x^2 - sinx$
    b) $y = e^x sinx$
    c) $y = 5e^x + cosx$
    d) $y = sinx\, cosx$

21. (1) State and prove first derivative test for local extrema.
    (2) Determine the values of constants $a, b, c \ and \ d$ so that $f(x) = ax^3 + bx^2 + cx + d$ has a local maximum at the point $(0,0)$ and a local minimum at the point $(-1,1)$.

22. (a) State the mixed derivative theorem.
    (b) Verify the mixed derivative theorem for $w = xy + \frac{e^y}{y^2+1}$.
    (c) Find the second-order derivatives of $w = \frac{x-y}{x^2+y}$.

(2×5=10 weightage)