24803757

QP CODE: 24803757

Reg No : ....

Name : ....

INTEGRATED MSC DEGREE EXAMINATION, JUNE 2024

Second Semester

INTEGRATED MSC BASIC SCIENCE-STATISTICS

CORE - IST2CR03 - AN INTRODUCTION TO PROBABILITY THEORY

2020 Admission Onwards

00E6E3F1

Time: 3 Hours | Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. What do you mean by simple and compound events?
2. Show that $P(\phi) = 0$.
3. An experiment involves tossing a single die. Let A be the event of observing a number greater than 2. Calculate the probability of A using probabilities of simple events.
4. Suppose that $P(A) = 0.4$, $P(B) = 0.3$ and $P(A \cap B) = 0.12$. Are events A and B independent?
5. State Bayes' rule
6. Consider the experiment of tossing a coin. Let X denote the number of heads obtained. Show that X is a random variable
7. Verify whether the following function is a probability density function, $f(x) = \begin{cases} 2x^3 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$
8. List the properties of joint probability mass function.
9. Verify that the following is a joint pdf for two dimentional continuous random variables(x,y)
   f(x,y) = $6x^2y$, 0< x< 1, o < y< 1
   = 0, otherwise
10. What is meant by conditional distribution of Y under the condition that X=x?

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Explain mn rule and extended mn rule with examples

12. State and prove additive law of probability

13. If $E_1, E_2, \ldots, E_n$ are n independent events with probabilities of occurrence $p_1, p_2, \ldots, p_n$, show that $P(E_1 \cup E_2 \cup \ldots \cup E_n) = 1 - (1 - p_1)(1 - p_2)\ldots(1 - p_n)$.

14. Let A and B be two events associated with an experiment and suppose $P(A) = 0.5$ while $P(A\, or\, B) = 0.8$. Let $P(B) = p$. For what values of p are a) A and B mutually exclusive b) A and B are independent

15. A random variable X has pmf given by P(X=1)=1/2, P(X=2)=1/3 and P(X=3)=1/6. Write down the distribution function of X

16. If X is a random variable with pdf $f(x) = \begin{cases} 1 & \text{if } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$. Find the pdf of $Y = -2log_e X$.

17. Given the joint pmf of X and Y is f(x,y)=1/4 for (x,y) =(0,0),(0,1),(1,0) ,(1,1). Obtain the marginal distributions of X and Y.

18. The joint pdf of a two-dimensional random variable (x,y) is given by ;
    f(x,y) = 2 ,for 0<x<1 and 0< y <x 0,
    = 0, Otherwise .
    Find the conditional density function of x given y

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a) Explain mutually exclusive events, equally likely events, exhaustive events and partitioning of sample space with examples b) For any two events A and B prove that $P((A \cap B^c) \cup (A^c \cap B)) = P(A) + P(B) - 2P(A \cap B)$.

20. State and prove Law of total probability. The probabilities of X, Y and Z becoming managers are 4/9, 2/9 and 3/9 respectively. The probabilities that the bonus scheme will be introduced if X, Y and Z becomes managers are 3/10, 1/2 and 4/5 respectively. What is the probability that Bonus scheme will be introduced?

21. a) The life length (in hours) of a certain brand of battery has the pdf $f(x) = \begin{cases} 100x^2 & x > 100 \\ 0 & \text{elsewhere} \end{cases}$. What is the probability that an arbitrarily chosen battery will last less than 200 hours if it is known that the battery is still functioning after 150 hours of service. b) X has the pdf given by $f(x) = bx(1-x), 0 \leq x \leq 1.$. Determine b so that $P(X < b) = P(X > b)$.

22. The joint probabilitydensty function of the two-dmensional variable (X,Y) is of the form:

$f(x,y)=ke^{(x+y)}, 0 \leq y < x < \infty$,

= 0, elsewhere

i)Determine the constant k

ii) Examine if X,Y are independent

(2×5=10 weightage)