QP CODE: 23800338

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INTEGRATED PG DEGREE EXAMINATION, DECEMBER 2023

Third Semester

INTEGRATED MSC BASIC SCIENCE-STATISTICS

Complementary - IST3CM04 - MATHEMATICS III- VECTOR CALCULUS , DIFFERENTIAL EQUATIONS AND LAPLACE TRANSFORM

2020 ADMISSION ONWARDS

D30C625C

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. If $\vec{r} = a\,cost\hat{i} + b\,sint\hat{j} + at\,tan\alpha\hat{k}$, Find $|\frac{d\vec{r}}{dt} \times \frac{d^2\vec{r}}{dt^2}|$

2. A particle moves so that its position vector is given by $\vec{r} = coswt\hat{i} + sinwt\hat{j}$. Show that $\vec{r} \times \vec{v}$ is a constant vector.

3. Define unit normal vector.

4. Solve $\frac{dy}{dx} = e^{2x+3y}$.

5. Define Homogeneous Functions.

6. Write Leibnitz's Linear Differential Equation.

7. Define Linear partial differential equations of the first order.

8. Solve $\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z} = \frac{du}{xyz}$.

9. Compute $L[e^{at}]$.

10. Write the First Shifting Property of the Laplace Transform.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. If $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$. Show that :
    a) $grad(\frac{1}{r}) = \frac{-\vec{r}}{r^3}$
    b) $\nabla r^n = nr^{n-2}\vec{r}$

12. Find the order and degree of the following differential equations:
    a) $\frac{d^2y}{dx^2} + 2(\frac{dy}{dx})^2 + y = 0$
    b) $y = x\frac{dy}{dx} + \frac{c}{\frac{dy}{dx}}$
    c)) $[1 + (\frac{dy}{dx})^2]^{\frac{3}{2}} = k\frac{d^2y}{dx^2}$
    d) $\frac{dy}{dx} = x^2 - 1$

13. Solve $\frac{dy}{dx} - x^2y = y^2e^{\frac{-1}{3}x^3}$.

14. Solve: $tany\frac{dy}{dx} = sin(x + y) + sin(x - y)$.

15. Find the order and degree of the partial differential equation $\frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x\, \partial t} + \frac{\partial^2 z}{\partial t^2} = 0$.

16. Form partial differential equation from the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. Also find the order and degree of that partial differential equation.

17. Find the Laplace Transform of $5\,cost + 5\,sin3t$.

18. Given that $L[\frac{sint}{t}] = tan^{-1}(\frac{1}{p})$, Find $L[\frac{sinat}{t}]$.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Find divergence and curl of the following vectors:
    a) $\vec{v} = xyz\hat{i} + 3x^2y\hat{j} + (xz^2 - y^2z)\hat{k}$
    b) $\vec{R} = (x^2 + yz)\hat{i} + (y^2 + zx)\hat{j} + (z^2 + xy)\hat{k}$

20. Check whether the differential equations are exact:
    a) $(1 + e^{\frac{x}{y}})dx + (1 - \frac{x}{y})e^{\frac{x}{y}}\,dy = 0$
    b) $(x^2 + y^2 - a^2)x\,dx + (x^2 - y^2 - b^2)y\,dy = 0$
    c) $(x^2y - 2xy^2)dx - (x^3 - 3x^2y)\,dy = 0$
    d) $(sec\,x\,tan\,y\,tan\,x - e^x)dx + (sec^2y\,secx)dy = 0$

21. Using Lagrange's Auxiliary equation , solve the following differential equations:
    a) $(mz - ny)p + (nx - lz)q = ly - mx.$
    b) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = xyz$

22. a) Using the parameter p, define the Laplace Transform.
    b) Find the Laplace Transform of 29 and t^{11}.

(2×5=10 weightage)