QP CODE: 23800336

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Reg No : .....

Name : .....

INTEGRATED PG DEGREE EXAMINATION, DECEMBER 2023

Third Semester

INTEGRATED MSC BASIC SCIENCE-STATISTICS

CORE - IST3CR02 - ESTIMATION THEORY

2020 ADMISSION ONWARDS

BFAC9A54

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. What is meant by Standard error?
2. Write down the sampling distribution of sample mean.
3. Write down the relationship between chi-square,F and t distribution.
4. Distinguish between point estimate and interval estimate.
5. Define Cramer-Rao inequality.
6. How do you show completeness?
7. Suppose $X_1, X_2, \cdots, X_n$ is a random sample from a Poisson distribution with mean λ. Find the moment estimator of λ.
8. What is the purpose of method of least squares?
9. Define Confidence interval.
10. Estimate the 95% confidence interval for the mean, based on 10 random samples 22,25,30,21,24,26,24,25,28,26 taken from normal population with standard deviation 5.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. A random sample of size 12 is taken from a normal population with mean 20 and variance 36. Find the probability that the sample variance s^2 will be less than population variance σ^2 ?

12. Derive the cumulative distribution for the r^{th} order statistic.

13. **i)** State and prove the invariance property of consistent estimator.
ii) Show that for a normal distribution, the sample mean is a consistent estimator of population mean.

14. i) Which are the desirable properties of a good estimator? Explain the terms.
ii) Explain MVUE.

15. Describe the procedure of maximum likelihood estimation and method of minimum variance.

16. Explain the method of minimum variance. Also, briefly explain the method of least squares.

17. Obtain the interval estimate for the difference between proportions of two binomial population.

18. A sample of size 12 is taken from $N(\mu, \sigma)$. Mean of sample is 10 and sample variance is 16. Find a 90% confidence interval for σ^2.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. i) Explain an example of a statistic following student's t distribution distribution with required proofs.

ii) If X is the random variable following F-distribution with (n_1,n_2) degrees of freedom then $\frac{1}{X}$ follows $F(n_2,n_1)$. Prove this.

20. A random sample (X_1,X_2,X_3,X_4,X_5) of size 5 is drawn from a normal population with unknown mean μ.
Consider the following estimators to estimate μ:

1) $t1 = \frac{X1 + X2 + X3 + X4 + X5}{5}$ 2) $t2 = \frac{X1 + X2}{2} + X3$ 3) $t3 = \frac{2X1 + X2 + \lambda X3}{3}$

where λ is such that t_3 is an unbiased estimator of μ.

i)Find λ .

ii) Are t_1 and t_2 unbiased?

iii) State giving reasons, the estimator which is best among t_1,t_2 and t_3.

21. i) Explain Bayesian estimation procedure.
ii) Let X follows b(n,p) and ∏(p)=1 for 0<p<1 be the prior distribution. Find the posterior distribution and mean of the posterior distribution.

22. Obtain the interval estimates for differences of means in case of small samples, when
i) σ_1 and σ_2 are known.
ii) σ_1 and σ_2 are unknown, but equal.

(2×5=10 weightage)