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Reg No   :   ....

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INTEGRATED MSC DEGREE EXAMINATION, JULY 2024

Fourth Semester

INTEGRATED MSC BASIC SCIENCE-STATISTICS

COMPLEMENTARY - IST4CM04 - MATHEMATICS IV - LINEAR ALGEBRA, THEORY OF EQUATIONS, SPECIAL FUNCTIONS, NUMERICAL METHODS

2020 Admission Onwards

826507B5

Time: 3 Hours   Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Let P be the m x m matrix that is obtained from the identity matrix I_m by permuting its rows in some way. Then prove that for any m x n matrix A the matrix PA is the matrix obtained from A by permuting its rows in precisely the same way.

2. Reduce to Hermite form of the matrix
$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

3. Explain consistent system of linear equations.

4. Explain algebraic multiplicitiy.

5. Multiply the roots of the equation $x^4 + 2x^3 + 4x^2 + 6x + 8 = 0$ by $\frac{1}{2}$.

6. Prove that $x^5 + x^3 + x + 1 = 0$ has excatly one real root?

7. Define Eulerian Integral of I^{st} Kind.

8. Explain the First and Second Forward Differences.

9. Write the relation between $\Delta, \ \nabla \ \& \ E$

10. The following table gives correspoding values of $x \ \& \ y$. Display the Forward difference table.

x	0	1	2	3	4
y	3	6	11	18	27

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Prove that the elementary row operations do not affect row rank.

12. If λ is an eigen value of $f : V \to V$ then prove that the geometric multiplicity of λ is less than or equal to the algebraic multiplicity of λ.

13. Expain the following Polynomials:
    a) Linear   b) Incomplete
    c) Quadratic   d) Real
    e) Cubic   f) Complete

14. Solve the equation : $4x^4 - 20x^3 + 33x^2 - 20x + 4 = 0$.

15. Prove that $\int_0^1 \frac{dx}{\sqrt{1-x^4}} = \frac{\sqrt{\Pi}}{4} \frac{\Gamma(\frac{1}{4})}{\Gamma(\frac{3}{4})}$.

16. Prove that $\frac{1}{2}\Gamma(\frac{1}{4})\Gamma(\frac{3}{4}) = \frac{\pi}{\sqrt{2}}$.

17. Given that

x	1	2	3	4	5
y	2	5	10	17	26

Find the value of $\nabla^2 y_5$.

18. Prove that $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1 + \frac{1}{4}\delta^2}$.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. For the matrix $\begin{bmatrix} 1 & 2 & 0 & 3 \\ 1 & 2 & 3 & 3 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$, determine the maximum number of linearly independent rows and the maximum number of linearly independent columns.

20. Solve by Cardon's method: $x^3 - 15x^2 - 33x + 847 = 0$

21. Find the value of $\Gamma(\frac{1}{2})$.

22. A slider in a machine moves along a fixed straight rod. Its distance $x \ cm$ along the rod is given below for various values of the time $t \ sec$. Find the velocity and acceleration of the slider when $t = 0.3 \ sec$

t	0	0.1	0.2	0.3	0.4	0.5	0.6
x	30.13	31.62	32.87	33.64	33.95	33.81	33.24

(2×5=10 weightage)