QP CODE: 24803626

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Reg No : ....

Name : ....

INTEGRATED MSC DEGREE EXAMINATION, JUNE 2024

Fifth Semester

INTEGRATED MSC BASIC SCIENCE-STATISTICS

CORE - IST5CR01 - REAL ANALYSIS I

2020 Admission Onwards

A25CB716

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Draw diagram in the plane of the cartesian product A× B for the given sets A={1,2,3}, B={xϵR: 1 ≤ x ≤ 3}
2. Let $A = B = \{x \epsilon R : -1 \leq x \leq 1\}$ and consider the subset $C = \{(x, y) : x^2 + y^2 = 1\}$ of A x B. Is this set a function. Justify your answer
3. State the principle of Mathematical induction.
4. Define finite set and infinite set.
5. If $0 < a < b$, show that $a < \sqrt{ab} < b$.
6. Define a divergent sequence.
7. Give two examples of monotone decreasing sequences.
8. Show that a Cauchy sequence of real numbers is bounded.
9. State Bolzano Intermediate value theorem.
10. State interior extremum theorem.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Let $g(x) = x^2$ and $f(x) = x + 2$ for $x \in R$ and let h be the composite function $h = g \circ f$. a) Find the direct image $h(E)$ of $E = \{x \in R : 0 \leq x \leq 1\}$ b) Find the inverse image $h^{-1}(G)$ of $G = \{x \in R : 0 \leq x \leq 4\}$

12. If f is a bijection of A onto B, show that f^{-1} is a bijection of B onto A

13. If S and T are sets and $T \subseteq S$ then show that a) If S is a countable set, then T is a countable set. b) If T is an uncountable set, then S is an uncountable set

14. Let the function f be defined by $f(x) = \frac{2x^2+3x+1}{2x-1}$ for $2 \leq x \leq 3$. Find a constant M such that $|f(x)| \leq M$ for all x satisfying $2 \leq x \leq 3$

15. Let $X = (x_n)$ and $Y = (y_n)$ be sequences of real numbers that converge to x and y respectively . Then show that the sequence XY converge xy.

16. State and prove monotone subsequence theorem.

17. Explain a) uniform continuity b) Nonuniform continuity criteria

18. State and prove the product rule of differentiation.

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. a) Explain composite functions
b) Let $f : A \to B$ and $g : B \to C$ be functions. (i) Show that if $g \circ f$is injective, then f is injective (i) Show that if $g \circ f$is surjective, then g is surjective

20. a) Show that if $I_n = [a_n, b_n]$, $n \in N$, is a nested sequence of closed , bounded intervals then there exists a number $\xi \in R$ such that $\xi \in I_n$ for all $n \in N$.
b) Show that if $I_n = [a_n, b_n]$, $n \in N$, is a nested sequence of closed , bounded intervals such that the lengths $b_n - a_n$ of I_n satisfy $inf\,\{b_n - a_n : n \in N\} = 0$, then number ξ contained in I_n for all $n \in N$ is unique

21. a) State and prove root test
b) State and prove ratio test

22. a) Let $A \subseteq R$, let f and g be functions on A to R, and let $r \in R$. Suppose that $c \in A$ and that f and g are continuous at c , then show that (i) $f + g, f - g, fg$ and rf are continuous at c (ii) If $h : A \to R$ is continuous at $c \in A$ and if $h(x) \neq 0$ for all $x \in A$, then the quotient $\frac{f}{h}$is continuous at c .
b) State and prove boundedness theorem

(2×5=10 weightage)