QP CODE: 24800577

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Reg No : .....

Name : .....

INTEGRATED MSC DEGREE EXAMINATION, DECEMBER 2023

Sixth Semester

NTEGRATED MSC BASIC SCIENCE-STATISTICS

CORE - IST6CR02 - COMPLEX ANALYSIS

2020 Admission Onwards

96247EB6

Time: 3 Hours Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Simplify $\frac{4+i}{2-3i}$
2. Define conjugate of a complex number
3. Find the real and imaginary parts of $f(z) = sin\, z$
4. Using the definition of limits show that $lim_{z \to z_0} \overline{z} = \overline{z_0}$
5. Define harmonic functions
6. Define contour integral
7. State Gauss's mean value theorem
8. State Taylor's theorem
9. Find the isolated singular points of the function $\frac{z+1}{z^3(z^2+1)}$
10. Show that $z_0 = 0$ is a removable singular point of the function $f(z) = \frac{1-\cos z}{z^2}$

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Express the the complex number $-1 - i$ in its polar form
12. Find the value of a) $(1+i)^4$ b) $(\sqrt{3}+i)^3$

13. Show that if a function $f(z)$ is continuous and nonzero at a point z_0, then $f(z) \neq 0$ throughout some neighborhood of that point.

14. If $f(z) = \cos hx \cos y + i \sin hx \sin y$ then show that f is entire

15. If $w(t) = u(t) + i\, v(t)$ is a complex valued function of a real variable t and $w'(t)$ exists then show that a) $\frac{d}{dt}w(-t) = -w'(-t)$ b) $\frac{d}{dt}[w(t)]^2 = 2w(t)w'(t)$

16. If C is a simple closed curve enclosing the origin, prove that $\frac{1}{2\pi\, i}\int_C \frac{e^{az}}{z^{n+1}}dz = \frac{a^n}{n!}$

17. Obtain the Laurent series expansion of $\frac{1}{(z-1)(z-2)}$ and specify the region where the expansion is valid

18. Evaluate $\int_C \frac{2z^2+z}{z^2-1}dz$ where C is $|z-1| = 1$

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Find the square roots of a) $2i$ b) $1 - \sqrt{3}i$ and express them in rectangular coordinates

20. Find the principal value of a) $(-i)^i$ b) $(i)^i$

21. a) State and prove Liouville's theorem
    b) State and prove fundamental theorem of algebra

22. Find the residues at poles of the function $f(z) = \frac{z-1}{(z^2+1)^2(z^2-4)}$

(2×5=10 weightage)