QP CODE: 24803038

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Reg No : ....

Name : ....

INTEGRATED MSC DEGREE EXAMINATION, MAY 2024

Seventh Semester

INTEGRATED MSC BASIC SCIENCE-STATISTICS

CORE - IST7CR02 - THEORY OF BIVARIATE AND MULTIVARIATE DISTRIBUTIONS

2020 Admission Onwards

07FE958A

Time: 3 Hours

Weightage: 30

Part A (Short Answer Questions)

*Answer any **eight** questions.*

*Weight **1** each.*

1. Define bivariate random variables.
2. Let X be an n-dimensioal random vector and the random vector Y be defined as Y= AX+b where A is fixed m×n matrix and b is a fixed m-dimensional vector. Show that $C_y = AC_x A^T$.
3. Write a note on quadratic form.
4. State the reproductive property of multivariate normal distribution.
5. Give two characterisations of multivariate normal distribution.
6. State the condition for independence of two quadratic forms.
7. Give the MLE of mean vector in a multivariate normal distribution.
8. Write a note on generalized variance.
9. Explain testing of equality of mean vectors of two multivariate normal distribution.
10. Explain how Fisher's linear discriminent function is related to Mahalanobis D^2.

(8×1=8 weightage)

Part B (Short Essay/Problems)

*Answer any **six** questions.*

*Weight **2** each.*

11. Let X, Y be two discrete random variables with joint pmf f(x,y) = x+2y/18 , x,y= 1,2 . Find the marginal pmf of X and Y.

12. Given the joint probability masses of X,Y as f(x,y) = 1/4 for (x,y) = (1,1), (1,2), (2,1), (2,2) . Examine whether X and Y are independent.

13. (i) Explain conditional mean. (ii) Let X and Y are two random variables with joint pmf f(x,y) = x+2y/18 , x=1,2 and y = 1,2. Find (a) marginal densities of x and y (b) E(X/Y=2)

14. (i) Define non singular multivariate normal distribution (ii) If X~ Np then show that the marginal distribution of any subset of q<p component of X follows Nq

15. Derive the marginal distribution of $x^{(1)}$ of q random variables of $x=(x^{(1)},x^{(2)}) \sim N_p(\mu,\Sigma)$

16. Derive the reproductive property of Wishart distribution.

17. Let A and Σ be partitioned in to $P_1, P_2,....P_q$ rows and columns $P_1+ P_2+....P_q = P$. Show that if $\Sigma_{ij}=0$ for i≠jand if A follows W(Σ,n), then $(A_{11},A_{22},......A_{qq})$ are independently distributed as $W(\Sigma_{jj},n)$.

18. Explain Uses of T^2

(6×2=12 weightage)

Part C (Essay Type Questions)

*Answer any **two** questions.*

*Weight **5** each.*

19. Given the joint density function of two random variables x and y as f(x,y) = 2-x-y, 0<x<1, 0<y<1 and 0 otherwise. Find
(i) Marginal densities of X and Y
(II) Conditional distribution of x given y and that of y given x
(iii) Conditional variance of y given x

20. Let X ~ Np(μ,Σ) suppose that X be partitioned as $\begin{pmatrix} x^{(1)} \\ x^{(2)} \end{pmatrix}$ and define $y^{(1)} = x^{(1)}+Mx^{(2)}$, $y^{(2)}= x^{(2)}$.
Determine M such that $y^{(1)}$ and $y^{(2)}$are independent . Hence find the conditional distribution.

21. (i) Derive the distribution of the sample mean vector (ii) If $X_1,X_2,.....X_N$ are N independent Xβ distributed according to N(μβ,Σ) and C = $(C\alpha\beta)_{Pxp}$ be an orthogonal matrix. Let Yα = $\sum_{\beta=1}^{n} C\alpha\beta X\beta$ then show that Yα's are normally and independently distributed.

22. Explain the procedure for the following (i) Testing of hypothesis about mean vector of multivariate normal distribution. (ii) Testing of equality of mean vectors two multivariate normal distribution.

(2×5=10 weightage)