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Name..................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, APRIL 2022

Fifth Semester

Core Course—MATHEMATICAL ANALYSIS

(Common for Model I and Model II B.Sc. Mathematics and B.Sc. Computer Applications)

[2013 to 2016 Admissions]

Time : Three Hours Maximum Marks : 80

Part A

Answer **all** *questions.*
Each question carries 1 mark.

1. Give an example of a set which is bounded above but not bounded below.
2. Find the infimum and supremum if it exists for the set $\left\{\frac{(-1)^n}{n}, n \in \mathrm{N}\right\}$.
3. State Archimedian property of real numbers.
4. Is Q, the set of rational numbers order complete.
5. What is the derived set of the set $\left\{1, -1, 1\frac{1}{2}, -1\frac{1}{2}, 1\frac{1}{3}, -1\frac{1}{3}, \dots\right\}$?
6. Define countable and uncountable sets.
7. Define a bounded sequence.
8. What is $\lim_{n\to\infty} n^{\frac{1}{n}}$?
9. If $z = 2 + 3i$, what is z^{-1} ?
10. Find Arg z, where $z = \frac{i}{-2-2i}$.

(10 × 1 = 10)

Part B

Answer any **eight** *questions.*
Each question carries 2 marks.

11. Show that the greatest member of a set, if it exists is the supremum of the set.
12. State Dedekind's form of completeness property.

Turn over

13. Find the smallest and greatest member of the set $\left\{\frac{1}{n}, n \in \mathrm{N}\right\}$ if they exist.

14. Define an open set is the set Q of rational numbers open in R

15. What is a perfect set ? Give an example.

16. Show that the set $\mathrm{S} = \{x : 0 < x < 1\}$ in R is open, but not closed.

17. Define the interior of a set. What is the interior of N and Q ?

18. Give an example of a sequence (a) which oscillates infinitely ; (b) which oscillates finitely.

19. Define a Cauchy sequence. Give an example.

20. Define a monotonic sequence. Give an example.

21. Locate $z_1 + z_2$ and $z_1 - z_2$ where $z_1 = -1 + 2i, z_2 = 1 + 4i$.

22. Find the square roots of $1 + \sqrt{3}i$ and express them in rectangular co-ordinates.

$(8 \times 2 = 16)$

Part C

*Answer any **six** questions.*
Each question carries 4 marks.

23. Show that the real number field is Archimedean.

24. Show that every open interval is an open set.

25. Show that a set is closed iff its complement is open.

26. Show that every convergent sequence is bounded.

27. Show that the sequence $\{\mathrm{S}_n\}$ where $\mathrm{S}_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$ cannot converge.

28. Show that $\lim \dfrac{(3n+1)(n-2)}{n(n+3)} = 3$.

29. Show that the sequence $\{\mathrm{S}_n\}$ where $\mathrm{S}_n = \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{n+n}$ is convergent.

30. Show that $\lim\limits_{n\to\infty}\left[\frac{1}{\sqrt{n^2+1}} + \frac{1}{\sqrt{n^2+2}} + \ldots + \frac{1}{\sqrt{n^2+4}}\right] = 1$.

31. Sketch the points determined by the condition (a) $\mathrm{Re}(\bar{z} - 1) = 2$; (b) $|2\bar{z} + i| = 4$.

$(6 \times 4 = 24)$

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) State and prove Bolzano-Weierstrass theorem, for sets.

    (b) Show that a countable union of countable sets is countable.

33. (a) State and prove Cauchy's general principle of convergence.

    (b) Use Cauchy's general principle of convergence to show that $\left\{\frac{n}{n+1}\right\}$ is convergent.

34. (a) Show that $[r^n]$ converges iff $-1 < r < 1$.

    (b) Show that a necessary and sufficient condition for the convergence of a monotonic sequence is that it is bounded.

35. (a) State and prove Cauchy's first theorem on limits.

    (b) Let $\{S_n\}$ be a sequence such that $S_{n+1} = 2 - \frac{1}{S_n}, n \geq 1$ and $S_1 = \frac{3}{2}$. Show that $\{S_n\}$ is bounded and monotonic and converges to 1.

(2 × 15 = 30)