E 2935   (Pages : 3)   Reg. No..............................

Name..................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, APRIL 2022

Fifth Semester

Core Course—DIFFERENTIAL EQUATIONS

(Common for Model I and Model II, B.Sc. Mathematics and B.Sc. Computer Applications)

[2013 to 2016 Admissions]

Time : Three Hours   Maximum Marks : 80

Part A

*Answer **all** questions.*
Each question carries 1 mark.

1. Write the general form of a homogeneous differential equation.

2. What is an exact differential equation ?

3. Write Bernoulli's differential equation.

4. Distinguish between general solution and particular solution.

5. Find the general solution of $\dfrac{d^2 y}{dx^2} + 25y = 0$.

6. Solve the equation $(D - 1)^2 y = 0$.

7. Write Bessel's function of the first kind of order $- n$.

8. Write the parametric representation of a surface.

9. Write the Lagrange's equation.

10. Write the order and degree of the equation $x\dfrac{\partial \theta}{\partial x} + y\dfrac{\partial \theta}{\partial y} + \dfrac{\partial \theta}{\partial t} = 0$.

$(10 \times 1 = 10)$

Part B

*Answer any **eight** questions.*
Each question carries 2 marks.

11. Find an integrating factor of the equation $\left(y - 2x^3\right)dx - x\,(1 - xy)\,dy = 0$.

12. Represent the family of all circles through the origin and tangent to the y-axis in the form $F(x, y, c) = 0$.

13. Test whether $\left(2x + e^y\right)dx + xe^y dy = 0$ exact.

Turn over

14. Solve the equation $y = p \sin p + \cos p$.

15. Represent the family of curves $e^x y = c$ by differential equations.

16. Solve the equation $\dfrac{d^2 y}{dx^2} + 2 \dfrac{dy}{dx} + 5y = 0$.

17. Find the particular solution of $\dfrac{d^2 y}{dx^2} - 13 \dfrac{dy}{dx} + 12y = e^{-2x}$.

18. Find the equation of the tangent plane to the surface at the point (x, y, z).

19. Form the partial differential equation by eliminating a and b from the equations $z = ax + by + a$.

20. Find the partial differential equation from $z = x + y + f(xy)$.

21. Define the terms ordinary point and regular singular point.

22. Express $J_5(x)$ in terms of $J_0(x)$ and $J_1(x)$.

$$(8 \times 2 = 16)$$

Part C

*Answer any **six** questions.*
Each question carries 4 marks.

23. Solve the equation $e^y dx + \left(xe^y + 2\right) dy = 0$.

24. Solve $y = p \tan p + \log \cos p$.

25. Find the orthogonal trajectories of $y = \sqrt{x + c}$.

26. Solve $\left(D^2 + 4\right) y = \cos 3x$.

27. Use the method of undetermined coefficients to solve $\dfrac{d^2 y}{dx^2} + 3 \dfrac{dy}{dx} + y = 3e^x$.

28. Show that $\dfrac{d}{dx}\left[x^{-n} J_n(x)\right] = -x^{-n} J_{n+1}(x)$.

29. Solve in series the equation $\dfrac{d^2 y}{dx^2} + xy = 0$.

30. Solve $a(p + q) = z$.

31. Solve $\left(\dfrac{y^2 z}{x}\right) p + xzq = y^2$.

$$(6 \times 4 = 24)$$

Part D

*Answer any **two** questions.*
Each question carries 15 marks.

32. (a) Solve by the method of variation of parameters $\dfrac{d^2y}{dx^2} + 4y = \tan 2x$.

    (b) Solve $x^2 \dfrac{d^2y}{dx^2} + x \dfrac{dy}{dx} - 4y = x$.

33. Obtain the power series solution of $x \dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - y = 0$.

34. (a) Show that $J_{1/2}(x) = \sqrt{\dfrac{2}{\pi x}} \cos x$.

    (b) Prove that $J_{5/2}(x) = \sqrt{\dfrac{2}{\pi x}} \left[\dfrac{3 - x^2}{x^2} \sin x - \dfrac{3}{x} \cos x \right]$.

35. Solve the equation $(y - z)\, p + (z - x)\, q = x - y$ and find the particular solution which passes through the parabola $y = x^2$, $x = 2$.

$(2 \times 15 = 30)$