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Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, APRIL 2022

Fifth Semester

Core Course (6)—Mathematical Methods—I

(For B.Sc. Statistics)

[2013—2016 Admissions]

Time : Three Hours Maximum Marks : 80

Part A (Short Answer Questions)

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Prove that between any two real numbers there is an irrational number.
2. Let E^1 be the set of limit point of a set E. Prove that E^1 is closed.
3. Suppose $\{s_n\}$ si monotonic. Then $\{s_n\}$ converges if and only if it is bounded.
4. Prove that $\lim_{n\to\infty} \frac{2n}{n+3} = 2$, by definition.
5. Define a Cauchy sequence.
6. For what values of x does the series $(1-x)+(x-x^2)+(x^2-x^3)+\ldots$ converge.
7. Does the series $\sum_{n=1}^{\infty} \frac{1+n}{1+n^2}$ converge.
8. State Rolle's theorem on a closed bounded interval.
9. Let $f(x)=x, 0\le x\le 1$ and p be the subdivision $\left\{0, \frac{1}{3}, \frac{2}{3}, 1\right\}$ of [0, 1]. Compute U (f ; p) and L(f ; p).
10. Define uniform convergence of sequences of functions.

(10 × 1 = 10)

Turn over

Part B (Brief Answer Questions)

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Prove that the union of any collection of open sets is an open set.
12. Show that every closed set in E_1 is there an intersection of a countable collection of open set.
13. Define countable and uncountable sets with examples.
14. Define a null sequence and give any two examples.
15. Show that sequence $\langle (-1)^n \rangle$ does not converge.
16. Prove that every Cauchy sequence is bounded.
17. Examine the convergence of the series :

$$1+\frac{1}{2^2}+\frac{2^2}{3^3}+\frac{3^3}{4^4}+\frac{4^4}{5^5}+\ldots.$$

18. Test for convergence of the series :

$$\frac{1}{\log 2}+\frac{1}{\log 3}+\frac{1}{\log 4}+\ldots.$$

19. Explain D-Alembert's ratio test.
20. Define continuity and uniform continuity.
21. Using definition of limit prove that $\underset{x\to a}{\text{Lt}}\ \frac{x^2-a^2}{x-a}=2a.$
22. Verify Rolle's Theorem for $f(x)=x(x-3)^2$ on [0, 3].

(8 × 2 = 16)

Part C (Descriptive/Short Essay Type Questions)

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. A set is closed if and only if its compliment is open.
24. Define (i) Isolated point ; (ii) Derived set ; (iii) Adherent point.
25. Show that every monotonically increasing sequence which is bounded above converge to its least upper bound.

26. Prove that $\underset{n\to\infty}{\text{Lt}} \left(\frac{n^n}{n!}\right)^{\frac{1}{n}} = e$.

27. Examine the convergence of the following series :

$$\sum_{n=1}^{\infty} \left(\sqrt{n^3+1} - \sqrt{n^3}\right).$$

28. Discuss the convergence of the series :

$$\Sigma \frac{1}{(\log \log n)^{\log n}}.$$

29. State Lagrange's mean value theorem.

30. Show that the function f defined by $f(x) = 2x^2 - 3x + 5$ is uniformly continuous on $[-2, 2]$.

31. Prove that $f(x) = 3x + 1$ is integrable on [1, 2] and $\int_1^2 (3x+1)\,dx = \frac{11}{2}$.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. State and prove Bolzano-Weistrass theorem.

33. State and prove Cauchy's first theorem on limits.

34. State and prove Cauchy's mean value theorem.

35. State and prove Fundamental theorem of integral Calculus.

(2 × 15 = 30)