E 3133

00003133

Reg. No.................................

Name....................................

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, APRIL 2022

Fifth Semester

THEORY OF ESTIMATION

(For B.Sc. Statistics)

(2013–2016 Admissions)

Time : Three Hours — Maximum Marks : 80

Part A (Short Answer Questions)

Answer **all** *questions briefly.*
Each question carries 1 *mark.*

1. What is MLE ?
2. Define complete sufficient Statistics.
3. Under what conditions least square estimator coincides with MLE ?
4. Define F-Statistic.
5. Give an example of an estimate which is consistent, but not unbiased.
6. Does the sample mean converge to population mean in Normal distribution.
7. Write the p.d.f. of a Chi-square distribution.
8. State Cramer-Rao inequality.
9. Write the interval estimate of Normal population mean in large sample.
10. Define confident coefficient.

$(10 \times 1 = 10)$

Part B (Brief Answer Questions)

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Obtain the MLE of Poisson parameter.
12. Show that the sample mean is unbiased for population mean.
13. Describe the relationship among '*t*', Chi-square and F distributions.

Turn over

14. State Factorisation theorem for sufficiency.

15. If X is a Chi-square variate with 8 degrees of freedom find its mean and variance.

16. Determine the relative efficiency of sample mean over sample median for $N(\mu, \sigma^2)$.

17. State the features of F distribution.

18. What are the properties of MLE ?

19. Briefly explain method of moments.

20. Let X_1, X_2 be a random sample of size 2 from N (0, 1). What is the distribution of $\frac{(X_1 + X_2)^2}{(X_1 - X_2)^2}$.

21. Derive the m.g.f. of a t distribution with 5 d.f.

22. Distinguish between parameter and statistic.

(8 × 2 = 16)

Part C (Short Essay Questions)

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. For a rectangular distribution over (a, b) find the MLEs of a and b.

24. For a distribution :

$$f(x,\theta) = \frac{1}{\theta} e^{-x/\theta} \; ; x \geq 0, \theta > 0$$
$$= 0, \text{otherwise}.$$

Show that $\bar{x}$ is consistent estimator of θ.

25. The diameter of a tube is normal with variance 0.09. A sample of 30 tubes has a mean 6.4 cms diameter. Find 95% confidence interval for the population mean.

26. State Rao-Blackwell theorem and mention any one application as example.

27. How large a sample is to be drawn from a Normal population with mean 16 and variance 9, if the sample mean is to lie between 14 and 18 with probability 0.95 ?

28. Obtain $100(1-\alpha)\%$ confidence interval for the variance of a normal population $N(\mu, \sigma^2)$.

29. Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample from population $f(x, \theta) = \frac{1}{2} e^{-|x/\theta|}$, find the MLE of θ. Is it unbiased for θ.

30. Establish the confidence interval for the difference of poportions of two binomial populations.

31. If T is a consistent estimator of θ then show that T^2 is consistent for θ^2.

(6 × 4 = 24)

Part D (Essay Questions)

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. Derive the confidence interval of μ in Normal population $N(\mu, \sigma^2)$ when (i) σ is known ; (ii) σ is unknown and sample size is small.

33. Derive 't' distribution stating its applications. What is the mean of this distribution ?

34. Define unbiased estimator. Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample of size n from a $B(1-p)$ and $T = \sum X_i$. Show that $\frac{T(T-1)}{n(n-1)}$ is an unbiased estimator of p^2.

35. Let $X_1, X_2, X_3, \ldots, X_n$ be a random sample from Cauchy distribution with $(\mu, 1)$. Show that the sample mean is not consistent for μ. Suggest a consistent estimator for it.

(2 × 15 = 30)