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Reg. No..................................

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B.A. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2022

Fourth Semester

Complementary Course—CALCULUS, EXPONENTIAL AND LOGARITHMIC FUNCTIONS

(For B.A. Economics)

(2013–2016 Admissions)

Time : Three Hours

Maximum Marks : 80

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. What is $\lim\limits_{x \to z}\left(x^4 + 3x\right)$?

2. If $y = -6x^5$, find $\dfrac{dy}{dx}$.

3. Define the term critical point.

4. If $f(x) = e^{g(x)}$, what is $f'(x)$?

5. Find the derivative of $2xe^x$.

6. If $y = (\log x)^2$, find $\dfrac{dy}{dx}$.

7. What is $\int a^{kx}\, dx$?

8. Find $\int \dfrac{1}{x}\, dx$.

9. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ for $z = e^x \sin y$.

10. Define income and cross price elasticities of demand.

$(10 \times 1 = 10)$

Turn over

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Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Find $\lim_{x \to 3} \left(5x^2 - 4x + 9\right)$.

12. Find the derivative of $y = (x+1)^2 (2x+3)$.

13. Test whether $y = 3x^2 - 14x + 5$ increasing, decreasing or remains stationary at $x = 4$.

14. At what rate will money quintuple if compounded continuously for 25 years ?

15. Find the first and second derivative of $y = \log_a 3x$.

16. Evaluate $\int x^{-1/5} dx$.

17. Find $\int \left(6e^{3x} - 8e^{-2x}\right) dx$.

18. State fundamental theorem of integral calculus.

19. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ for $z = 5x^3 - 3x^2 y^2 + 7y^5$.

20. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ when $z = \left(x^3 + 7y^2\right)^4$.

21. If $z = (12x - 7y)^2$ find z_{xy} and z_{yx}.

22. If $z = x^{0.7} y^{0.2}$ find z_{xx} and z_{yy}.

$(8 \times 2 = 16)$

Part C

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. Use implicit differentiation to find $\dfrac{dy}{dx}$ from the equation $3x^4 - 7y^5 - 86 = 0$.

24. Test whether $f(x) = \dfrac{x-3}{x^2-9}$ continuous at $x = 3$.

25. Find the critical values and relative extrema if any of the function $f(x) = 2x^4 - 16x^3 + 32x^2 + 5$.

26. Find the effective annual interest rate on $\$\,100$ at 6 percent compounded :

    (a) Semiannually ; and (b) Continuously.

27. Find $\dfrac{dy}{dx}$, where $y = \ln\dfrac{x^3}{(2x+5)^2}$?

28. Find the integral for $y = \int \left(x^{\frac{1}{2}} + 3x^{-\frac{1}{2}} \right) dx$, given the initial conditions $y = 0$, when $x = 0$.

29. Integrate $\displaystyle\int_{0}^{3} \dfrac{6x}{x^2+1}\,dx$.

30. Find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$, where $z = \dfrac{6x+7y}{5x+3y}$?

31. A firm producing two goods x and y has the profit function :

    $\pi = 64x - 2x^2 + 4xy - 4y^2 + 32y - 14$.

    Find the profit maximising level of output for each of the two goods and test that the profits are maximised.

    $(6 \times 4 = 24)$

Turn over

Part D

*Answer any **two** questions.*
Each question carries 15 marks.

32. (a) Given $f(x) = g(x) \cdot h(x)$, where $g'(x)$ and $h'(x)$ both exist. Prove that :

$$f'(x) = g(x) \cdot h'(x) + h(x) \cdot g'(x).$$

   (b) Maximise the profit π for a firm given total revenue $R = 4000Q - 33Q^2$ and total cost

   $C = 2Q^3 - 3Q^2 + 400Q + 5000$, assuming $Q > 0$.

33. (a) Find the future value of a principal of $\$ 2000$ compounded semi annually at 12 percent for

   3 years using (i) an exponential function ; (ii) the equivalent natural exponential function.

   (b) Using logarithmic differentiation find the derivative of $g(x) = \dfrac{\left(5x^3 - 8\right)\left(3x^4 + 7\right)}{\left(9x^5 - 2\right)}$.

34. (a) Evaluate $\displaystyle\int \dfrac{6x^2 + 4x + 10}{\left(x^3 + x^2 + 5x\right)^3} \, dx.$

   (b) Find the area between the curves $y_1 = x^2 - 4x + 8$ and $y_2 = 2x$ from $x = 0$ to $x = 3$.

35. Optimize $z = 4x^2 + 3xy + 6y^2$

   subject to the constraint $x + y = 6$ using the method of Lagrange multipliers.

   $(2 \times 15 = 30)$