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Reg. No.....

Name.....

B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, NOVEMBER 2022

Fourth Semester

Core Course—VECTOR CALCULUS, THEORY OF EQUATIONS AND NUMERICAL METHODS

(Common for B.Sc. Mathematics Model I, II and B.Sc. Computer Applications)

(2013—2016 Admissions)

Time : Three Hours Maximum Marks : 80

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Find the parametric equation of the line through P (1, 2, – 1) and Q (–1, 0, 1).

2. If $f(x, y) = y - x$. Find the gradient ∇f at (2, 1).

3. Write the formula for the unit tangent vector of a differentiable curve $r(t)$.

4. Define a conservative field.

5. Write the formula for calculating surface area.

6. State Stoke's theorem.

7. Write an equation whose roots are two times the roots of the equation $x^3 - x^2 + x + 1 = 0$.

8. What is a standard reciprocal equation ?

9. If α, β, γ are the roots of the equation $x^3 - px^2 + qx - r = 0$ find $\Sigma\beta^2 r^2$ in terms of the co-efficients.

10. What is a transcedental equation ? Give an example.

(10 × 1 = 10)

Turn over

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Find the length of the curve $r(t) = (6 \sin 2t)i + (6 \cos 2t)j + 5tk, 0 \le t \le \pi$.

12. Find the unit normal vector N for the helix $r(t) = (a \cos t)i + (a \sin t)j + btk, a, b \ge 0, a^2 + b^2 \ne 0$.

13. Find the torsion of the curve $r(t) = \left(\frac{t^3}{3}\right)i + \left(\frac{t^2}{2}\right)j, t > 0$.

14. Find the tangent plane to the surface $z = x \cos y - ye^x$ at $(0, 0, 0)$.

15. Find the line integral of $f(x, y, z) = x + y + z$ over the straight line segment from (1, 2, 3) to (0, – 1, 1).

16. Find the circulation of the field $F = (x - y)i + xj$ around the circle $r(t) = (\cos t)i + (\sin t)j, 0 \le t \le 2\pi$.

17. Solve $x^3 - 7x^2 + 36 = 0$, given that the difference between two roots is 5.

18. If α, β, γ are the roots of $ax^3 + bx^2 + cx + d = 0$, find the values of :

    (i) $\Sigma(\alpha + \beta)^2$.

    (ii) $\Sigma\frac{\alpha}{\beta\gamma}$.

19. Find the equation each of whose roots is greater by unity than a root of $x^3 - 5x^2 + 6x - 3 = 0$.

20. If α, β, γ are the roots of $x^3 + qx + r = 0$, form the equation whose roots are $\alpha^3, \beta^3, \gamma^3$.

21. Explain bisection method of solving algebraic and transcendental equations.

22. Write the Newton-Raphson formula.

(8 × 2 = 16)

Part C

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. Show that the curvature of a circle of radius a is $\frac{1}{a}$.

24. Write the acceleration a in the form $a_T T + a_N N$ without finding T and N for the curve $r(t) = (a\cos t)i + (a\sin t)j + b + k$.

25. Find the point on the curve $r(t) = (12\sin t)i - (12\cos t)j + 5tk$ at a distance of 13π units along the curve from the origin in the direction opposite to the direction of increasing arc length.

26. Find the work done by $F = xyi + yzj + xzk$ from (0, 0, 0) to (1, 1, 1) over the path $r(t) = ti + t^2 j + t^4 k, 0 \le t \le 1$.

27. Find a potential function for the field $F = 2xi + 3yj + 4zk$.

28. Use Green's theorem to find the counter clockwise circulation and outward flux for the field $F = (y^2 - x^2)i + (x^2 + y^2)j$ over the triangle bounded by $y = 0$, $x = 3$ and $y = x$.

29. Solve the equation $x^4 - 12x^3 + 49x^2 - 78x + 40 = 0$ by removing its second term.

30. Solve the equation :

$x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$.

31. Use bisection method to find a root of $x^4 - x - 10 = 0$ correct to two decimal places.

(6 × 4 = 24)

Turn over

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. a) Find the directional derivative of $f(x, y, z) = x^2 + 2y^2 - 3z^2$ at (1, 1, 1) in the direction of $A = i + j + k$.

b) Estimate how much value of $f(x, y, z) = xe^y + yz$ will change if the point $P(x, y, z)$ moves 0.1 unit from $p_0(2, 0, 0)$ straight toward $p_1(4, 1, -2)$.

33. a) Find the area of the surface cut from the bottom of the paraboloid $x^2 + y^2 - z = 0$ by the plane $z = 4$.

b) Use Stoke's theorem to calculate the circulation of the field $F = 2yi + 3xj - z^2k$ around the circle $x^2 + y^2 = 9$ in the xy-plane, counterclockwise when viewed from above.

34. a) Solve by Cardan's method $x^3 + x^2 - 9x + 12 = 0$.

b) Solve by Ferrari's method $x^4 + 8x^3 + 6x^2 - 8x - 7 = 0$.

35. a) By Newton's method find a root of the equation $x^3 - 5x^2 + 4x - 3 = 0$ correct to three decimal places.

b) Use Newton-Raphson method to find a root of the equation $x \sin x + \cos x = 0$ correct to three decimal places, which is near $x = \pi$.

(2 × 15 = 30)