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Reg. No.....

Name.....

B.A. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2024

Fourth Semester

Complementary Course—CALCULUS - EXPONENTIAL AND LOGARITHMIC FUNCTIONS

(For B.A. Economics)

[2013–2016 Admissions]

Time : Three Hours Maximum Marks : 80

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. What is $\lim_{x \to 5} 9$?
2. Find $f'(x)$ if $f(x) = 3x^{-2}$.
3. What do you mean by the term inflection point ?
4. State natural logarithmic function rule.
5. If $y = e^{2x+1}$, what is $y'(x)$.
6. If $y = a^{5x^2}$, find $\frac{dy}{dx}$.
7. What is $\int \frac{1}{3x} dx$?
8. Find $\int_1^3 \left(4x^3 + 6x\right) dx$.
9. If $z = 9x^2y + 6xy^3$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.
10. Write the degree of the homogeneous function $z = x^3 + 2xy^2 + y^3$.

(10 × 1 = 10)

Turn over

Part B

Answer any **eight** *questions.*
Each question carries 2 *marks.*

11. Find $\lim_{x \to 6} \frac{4x^2 - 2x - 8}{5x^2 + 12}$.

12. Find the derivative of $y = \frac{(x+1)^2}{x^2 - 2x + 2}$.

13. Test whether the function $y = x^3 - 7x^2 + 6x - 2$ is increasing, decreasing or remains stationary at $x = 4$.

14. How long will it take money to double at 5 % interest when compounded quarterly ?

15. If $f(x) = \log_a \left(2x^2 + 1\right)$. Find $f'(x)$.

16. Evaluate $\int 16e^{-4x} dx$.

17. Evaluate $\int (x-9)^{7/4} dx$.

18. Show that $\int_5^{-5} (2x+3)dx = 0$.

19. Evaluate $\int_1^4 \left(x^{-1/2} + 3x^{1/2}\right)dx$.

20. Find Z_x and Z_y for $Z = (x + y)^2$.

21. If $Z = x^2 + xy + y^2$ find Z_{xy} and Z_{xx}.

22. If $Z = x^{0.4}y^{0.6}$ find Z_{xx} and Z_{yy}.

$(8 \times 2 = 16)$

Part C

Answer any **six** *questions.*
Each question carries 4 *marks.*

23. Test whether $f(x) = \frac{x^2 + 3x + 12}{x - 3}$ continuous at $x = 4$.

24. Find the derivative of $y = \frac{(8x-5)^2}{7x+4}$.

25. Find the relative extrema for the function $f(x) = -7x^2 + 126x - 25$ by (a) finding critical values and (b) determining if at the critical values the function is at a relative maximum or minimum.

26. Given a principal P of \$1000 at 6 % interest for 3 years, find the future value 3 when the principal is compounded (a) annually ; (b) quarterly.

27. Given that $y = \ln\left(x^2 - 6x + 10\right)$. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

28. Find $\int x^4\left(2x^5 - 5\right)^4 dx$.

29. Use substitution method to integrate $\int_0^3 8x\left(2x^2 + 3\right)dx$.

30. Given $Z = \left(x^3 + 7y^2\right)^4$. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.

31. Find the critical values for minimizing the cost of a firm producing two goods x and y when the total cost function is $c = 8x^2 - xy + 12y^2$ and the firm is bound by contract to produce a minimum combination of goods totalling 42, that is subject to the constraint $x + y = 42$.

(6 × 4 = 24)

Part D

Answer any **two** *questions.*
Each question carries 15 *marks.*

32. (a) For the function $y = -(x-8)^4$ (i) find the critical values (ii). Test whether the function has a relative maximum, relative minimum or inflection at critical points.

(b) Optimize $y = 7x^2 + 112x - 54$ at critical values and test for relative maximum or minimum at critical points.

33. (a) An animal population goes from 3.5 million in 1997 to 4.97 million in 2001. Express the population growth p in terms of a natural exponential function and determine the rate of growth.

(b) Find the slope of $y = \ln^2(x+4)$ at $x = 6$.

Turn over

34. (a) Evaluate $\int \frac{3x^2+2}{4x^3+8x}\,dx$.

(b) Draw the graph of the following functions and evaluate the area between $y_1 = 7 - x$ and $y_2 = 4x - x^2$ from x = 1 to x = 4.

35. (a) Optimize $f(x,y) = 26x - 3x^2 + 5xy - 6y^2 + 12y$ subject to 3x + y = 170.

(b) What combination of goods x and y should a firm produce to minimize cost when the join cost function is $c = 6x^2 + 10y^2 - xy + 30$ and the firm has a production quota of $x + y = 34$.

(2 × 15 = 30)