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# B.Sc. DEGREE (C.B.C.S.S.) EXAMINATION, MAY 2024

**Fourth Semester**

Complementary Course—Mathematics

FOURIER SERIES, DIFFERENTIAL EQUATIONS, NUMERICAL ANALYSIS AND ABSTRACT ALGEBRA

(For the programme B.Sc. Physics/Chemistry/Petrochemicals/Geology/Food Science and Quality Control and Computer Maintenance and Electronics)

[2013–2016 Admissions]

Time : Three Hours Maximum Marks : 80

## Part A

*Answer* **all** *questions.*
*Each question carries* 1 *mark.*

1. Find the fundamental periods of $\cos 2x, \cos 2\pi x$ and $\sin 2x$.
2. Write the Fourier series of a function $f(x)$ with period 2L by defining the Fourier co-efficient.
3. Write the power series representing the functions $\cos x$ and $\frac{1}{1-x}$, for $|x|<1$.
4. Find the radius of convergence of $\sum_{m=0}^{\infty} \frac{x^m}{c^m}, c \neq 0$.
5. Distinguish between linear and non-linear partial differential equation.
6. Form a partial differential equation by eliminating $a$ and $b$ from the equation $z = ax + y$.
7. Round off to four significant figures the numbers 30.0567 and 0.859378.
8. Is set $\mu_{m \times n}(R)$ of all $m \times n$ matrices under matrix multiplication a group. Justify your answer.
9. Define a cyclic group. Give an example.
10. Define a division ring.

$(10 \times 1 = 10)$

**Turn over**

## Part B

*Answer any* **eight** *questions.*
*Each question carries 2 marks.*

11. Define Bessel function of the first kind of order $n$.

12. Show that $\Gamma(\upsilon+1)=\upsilon\Gamma(\upsilon)$.

13. Write Rodrigue's formula for Legendre polynomials.

14. Form a partial differential equation by eliminating the arbitrary function $f$ from the equation $x+y+z=f\left(x^2+y^2+z^2\right)$..

15. Find the differential equation of all spheres of radius $r$, having centre in the $xy$ plane.

16. If $u = 3v^7 - 6v$, find the percentage error in $u$ at $v = 1$, if the error in $v$ is 0.05.

17. Find the product of the numbers 56.54 and 12.4 which are both correct to the significant digits given.

18. Find the number of terms of the exponential series such that their sum gives the value of $e^x$ correct to five decimal places for all values of $x$ in the range $0 \le x \le 1$.

19. If G is a group with binary operation *, then show that the left and right cancellation laws hold in G.

20. Prove that every cyclic group is abelian.

21. Given that $\sigma=\begin{pmatrix}1&2&3&4&5&6\\3&1&4&5&6&2\end{pmatrix}$ and $\tau=\begin{pmatrix}1&2&3&4&5&6\\2&4&1&3&6&5\end{pmatrix}$. Find $\tau^2\sigma$.

22. Determine whether {(1, 1, 0), (1, 0, 1), (0, 1, 1)} is a basis for $R^3$ over R.

(8 × 2 = 16)

## Part C

*Answer any* **six** *questions.*
*Each question carries 4 marks.*

23. Find the Fourier co-efficients of the function $f(x)$ given by

$$f(x)=\begin{cases}-k, & \text{if } -\pi<x<0\\ k, & \text{if } \ 0<x<\pi\end{cases}$$

and $f(x+2\pi)=f(x)$.

24. Solve by power series method the equation $y'=2xy$.

25. Show that $J_2'(x) = \frac{1}{2}\left[J_1(x) - J_3(x)\right]$.

26. Solve the equation $\left(y^2 z / x\right) p + xzq = y^2$.

27. Using bisection method obtain a root correct to three decimal places for the equation $x^3 - 3x - 5 = 0$.

28. Using false position method obtain a root of $x^3 - x - 1 = 0$, correct to 3 decimal places.

29. Let R be a ring with identity 0, show that for any $a, b \in R$ :

(a) $0a = a_0 = 0$ ; (b) $a(-b) = (-a)b = -(ab)$ ; (c) $(-a)(-b) = ab$.

30. Describe all ring homomorphisms of $z$ into $z$.

31. Show that any two bases of a finite dimensional vector space V over F have the same number of elements.

(6 × 4 = 24)

## Part D

*Answer any* **two** *questions.*
*Each question carries* 15 *marks.*

32. (a) Find the Fourier series for the function $f(x) = \begin{cases} 1 + x, & \text{if } -1 < x < 0 \\ 1 - x, & \text{if } \ \ 0 < x < 1, \end{cases}$ with $p = 2L = 2$.

(b) Find the Fourier Sine and cosine series for the function $f(x) = 2 - x, 0 < x < 2$.

33. Solve $x\left(y^2 + z\right)p - y\left(x^2 + z\right)q = z\left(x^2 - y^2\right)$ and find the surface that satisfies the above partial differential equation that contains the straight line $x + y = 0$, $z = 1$.

34. (a) Use iteration method to find correct to four significant figures a real root of $e^{-x} = 10x$.

(b) Use Newton-Raphson method to obtain a root correct to 3 decimal places of the equation $x - \cos x = 0$.

35. (a) Prove that every group is isomorphic to a group of permutations.

(b) Let $S = \{\alpha_1, \alpha_2, \ldots, \alpha_r\}$ be a finite set of linearly independent vectors of a finite dimensional vector space V over a field F. Show that S can be enlarged to a basis for V over F. Also show that if $B = \{B_1, B_2, \ldots, B_n\}$ is any basis for V over F, then $r \leq n$.

(2 × 15 = 30)